

Fixed Income Division

**NOMURA**

## **Lévy simple structural models**

Martin Baxter

Fixed Income Quantitative Research

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# Outline of talk

- Motivation
- Levy processes to model single-name credits
- Correlated Levy processes for basket models
- Comparison of different Levy models
- Calibration results

# Motivation and intuition

- Credit is about extreme events
- Extreme events through jumps of Levy process
- Jumps can be global or idiosyncratic
- Need a dynamic model, arbitrage-free
- Tractable

# Levy processes

- *Poisson process* – jumps of size 1 occur at rate  $\lambda$
- *Levy process* – jumps of size  $x$  occur at rate  $\nu(x)dx$
- Formally  $X(t)$  is a Levy process if it has stationary independent increments, that is
  - $X(s+t) - X(s)$  is independent of  $(X(u): u \leq s)$ , and is distributed as  $X(t)$ .
- All Levy processes are made up of drift, Brownian motion and jumps

$$E(\exp(\theta X_t)) = \exp(t \psi(\theta))$$

$$\psi(\theta) = \mu\theta + \frac{1}{2}\sigma^2\theta^2 + \int (e^{\theta x} - 1)\nu(x)dx$$

- Drift  $\mu$ , Brownian volatility  $\sigma$ , jump measure  $\nu$ .

# Gamma process

- *Gamma process* – pure jump increasing process with jump measure

$$\nu(x) = \gamma x^{-1} \exp(-\lambda x)$$

- Marginal distribution is the continuous gamma distribution.
- Parameter  $\gamma$  is jump intensity
- Parameter  $\lambda$  is inverse jump size
- Notation  $X(t) = \Gamma(t, \gamma, \lambda)$

- *Variance Gamma process* (Moosbrucker) – difference of two Gammas

$$X(t) = VG(t; \gamma, \lambda, \lambda_u) = \Gamma(t; \gamma, \lambda_u) - \Gamma(t; \gamma, \lambda)$$

# Levy credit model – basic Gamma model

- Structural model with value of the firm  $S(t)$

$$S_t = S_0 \exp(-\Gamma(t; \gamma, \lambda) + \mu t), \quad \mu = \gamma \log(1 + \lambda^{-1})$$

- Assume  $S(t)$  is a positive martingale with up-drift and down-jumps – “No news is good news”
- Entity defaults when  $S(t)$  goes below a threshold
- Reduces to log-normal as  $\gamma$  goes to infinity
- Reduces to pure intensity model for zero  $\gamma$
- Parameter  $\gamma$  is the “normalness” of the name

# Levy credit model - calculations

- Working with the transformed process

$$Y_t = \Gamma(\gamma; t, \lambda) + \mu(T - t)$$

- We can use either the European approximation

$$P(Y_T > k) = P(\Gamma(\gamma T) > \lambda k)$$

- Or the barrier probability

$$P(\sup_{t \leq T} Y_t > k) \cong P(Y_T > k) + f_{Y(T)}(k) \mu T \left( \frac{\phi + \exp(-\phi) - 1}{\phi^2} \right)$$

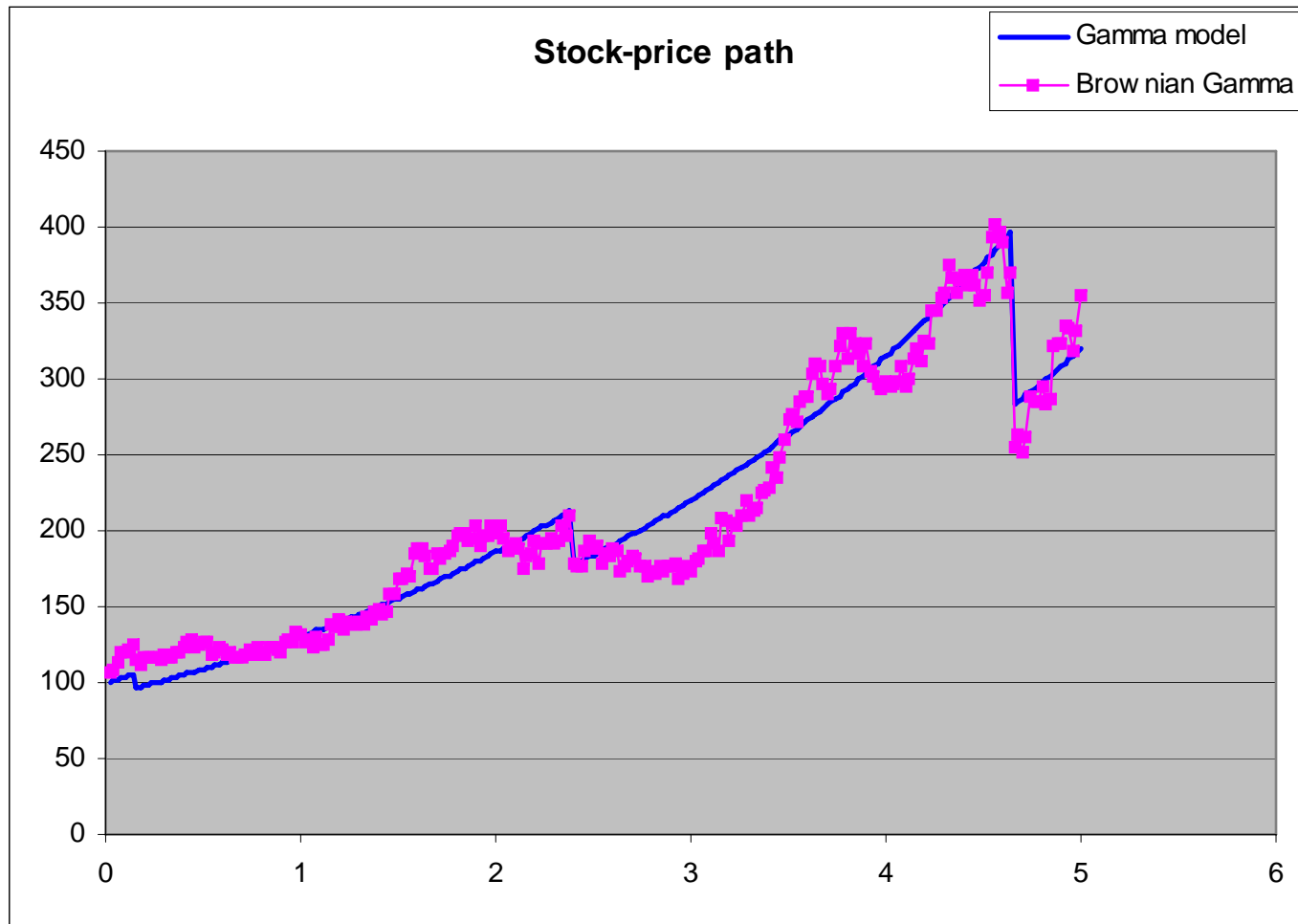
- Where

$$\phi = \gamma T (1 + \log c_0), \text{ and } c_0 = \frac{\gamma k}{\mu} \log \left( \frac{k}{\mu T} \right)$$

# Levy modelling – extra features

- Add “good news” jumps with Variance Gamma instead of Gamma (VG).
- Add “catastrophe” term with extra low-intensity high-impact global factor (CG).
- Add Brownian term to get continuous random movement as well (BG).
- Also various combinations, such as Brownian-Variance-Gamma (BVG), Catastrophe-Variance-Gamma (CVG), etc.

# Levy modelling – sample path



# Correlated Levy processes

- Start with any Levy process distribution  $X(t)$ .
- Make independent copies  $X_g$  (global factor) and  $\tilde{X}_1, \dots, \tilde{X}_n$  (idiosyncratic factors)
- Then define correlated Levy processes as

$$X_i(t) = X_g(\phi t) + \tilde{X}_i((1 - \phi)t)$$

- So that each  $X(i)$  has the same distribution as  $X$ , and the correlation between  $X(i)$  and  $X(j)$  is  $\phi$ .

# Levy models for CDO Pricing

- We can reformulate the Gamma model as

$$X_t^i = -\Gamma_g(t; \phi\gamma, \lambda) - \Gamma_i(t; (1-\phi)\gamma, \lambda)$$

- Decomposes the log-value into global and idiosyncratic Gamma terms
- Default by  $t$ , if  $X^i(t)$  below a threshold (European approximation)
- Lambda parameter is redundant due to scaling
- Phi parameter moves spread from junior to senior – equivalent to average Base Corr level
- Gamma parameter moves spread from equity/senior to mezzanine – controls slope of the Base Corr curve

# Linking back to Gaussian copula

- The Gaussian copula can be thought of as

$$X_i = \sqrt{\rho}Z_g + \sqrt{1-\rho}Z_i$$

- This can be made dynamic as

$$X_i(t) = \sqrt{\rho}W_g(t) + \sqrt{1-\rho}W_i(t)$$

- And then by the space-time equivalence of BM

$$X_i(t) = W_g(\rho t) + W_i((1-\rho)t)$$

- This now matches our general form.

# Comparison of Levy models

Model	CDX 5y	CDX 7y	CDX 10y	iTraxx 5y	iTraxx 7y	iTraxx 10y	Average (bp)
Cat Gamma	1.4	7.9	15.4	1.1	7.0	8.7	6.9
Variance Gamma	2.9	9.6	15.7	2.9	9.6	7.0	8.0
<b>Gamma</b>	<b>3.3</b>	<b>7.7</b>	<b>17.2</b>	<b>3.2</b>	<b>6.8</b>	<b>17.0</b>	<b>9.2</b>
Brownian Gamma	4.7	11.1	18.3	3.9	9.2	13.8	10.2
Brownian VG	2.8	21.9	44.2	2.3	18.2	40.6	21.7
Cat VG	1.4	28.6	48.1	1.0	26.4	34.7	23.4
Global Info Time	77.0	28.5	66.1	38.0	32.3	49.5	48.6
Gaussian copula	38.9	66.1	76.3	33.6	75.7	83.9	62.4

Both Gamma and its simple variants (CG, VG, and BG) do well.

BVG and CVG have symmetric up and down jumps which are not as good

Data weekly observed from 12 October 2005 to 5 April 2006.

# Calibration to CDO prices (CDX)

Tranche	5y CDX S7		7y CDX S7		10y CDX S7	
	Market	Model	Market	Model	Market	Model
0% - 3%	1346.1	1347.4	1613.7	1615.0	1791.9	1787.9
3% - 7%	107.0	105.9	248.0	246.8	542.5	546.8
7% - 10%	23.0	32.1	56.0	60.7	122.5	122.7
10% - 15%	11.0	17.7	24.5	31.8	58.8	54.1
15% - 30%	5.0	7.5	7.8	12.9	16.5	22.7
30% - 100%	2.5	0.9	3.5	1.5	4.5	3.1
Best fit score (bp)		<b>4.8</b>		<b>4.3</b>		<b>4.0</b>

Fitting as at 22 September 2006.

Maturity	CDX S7	
	Gamma	Phi
5y	29.0%	12.9%
7y	23.2%	14.9%
10y	12.1%	16.5%

# Calibration to CDO prices (iTraxx)

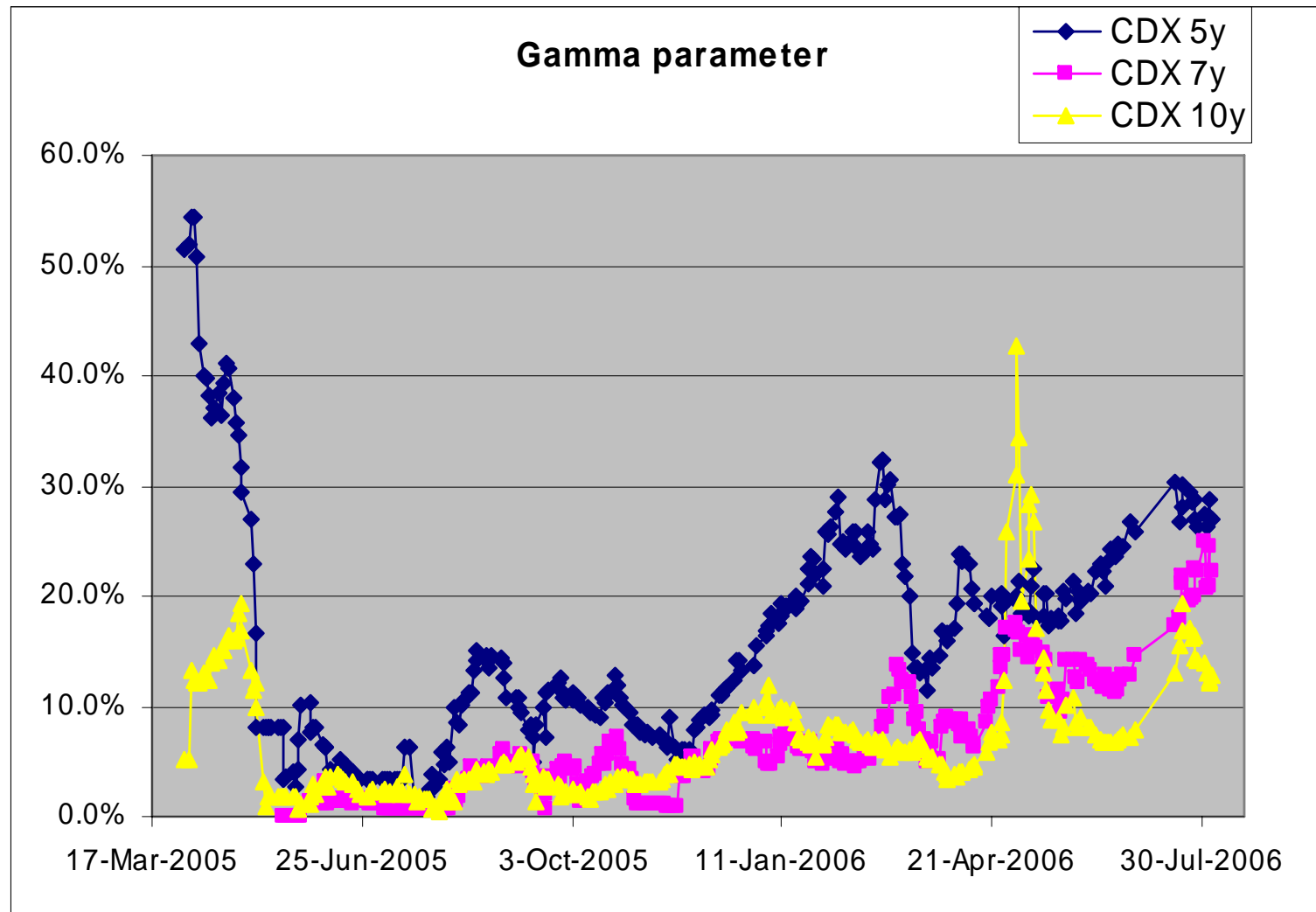
Tranche	5y iTraxx S6		7y iTraxx S6		10y iTraxx S6	
	Market	Model	Market	Model	Market	Model
0% - 3%	979.3	979.0	1285.7	1286.2	1455.7	1476.1
3% - 6%	74.0	67.4	185.0	180.8	460.0	405.1
6% - 9%	22.5	25.2	54.0	54.3	113.5	107.6
9% - 12%	10.5	14.9	26.0	30.6	52.5	56.1
12% - 22%	4.3	7.7	9.0	15.1	18.5	28.8
22% - 100%	1.3	1.0	2.5	1.9	4.1	4.3
Best fit score (bp)		<b>3.7</b>		<b>3.6</b>		<b>24.4</b>

Fitting as at 22 September 2006.

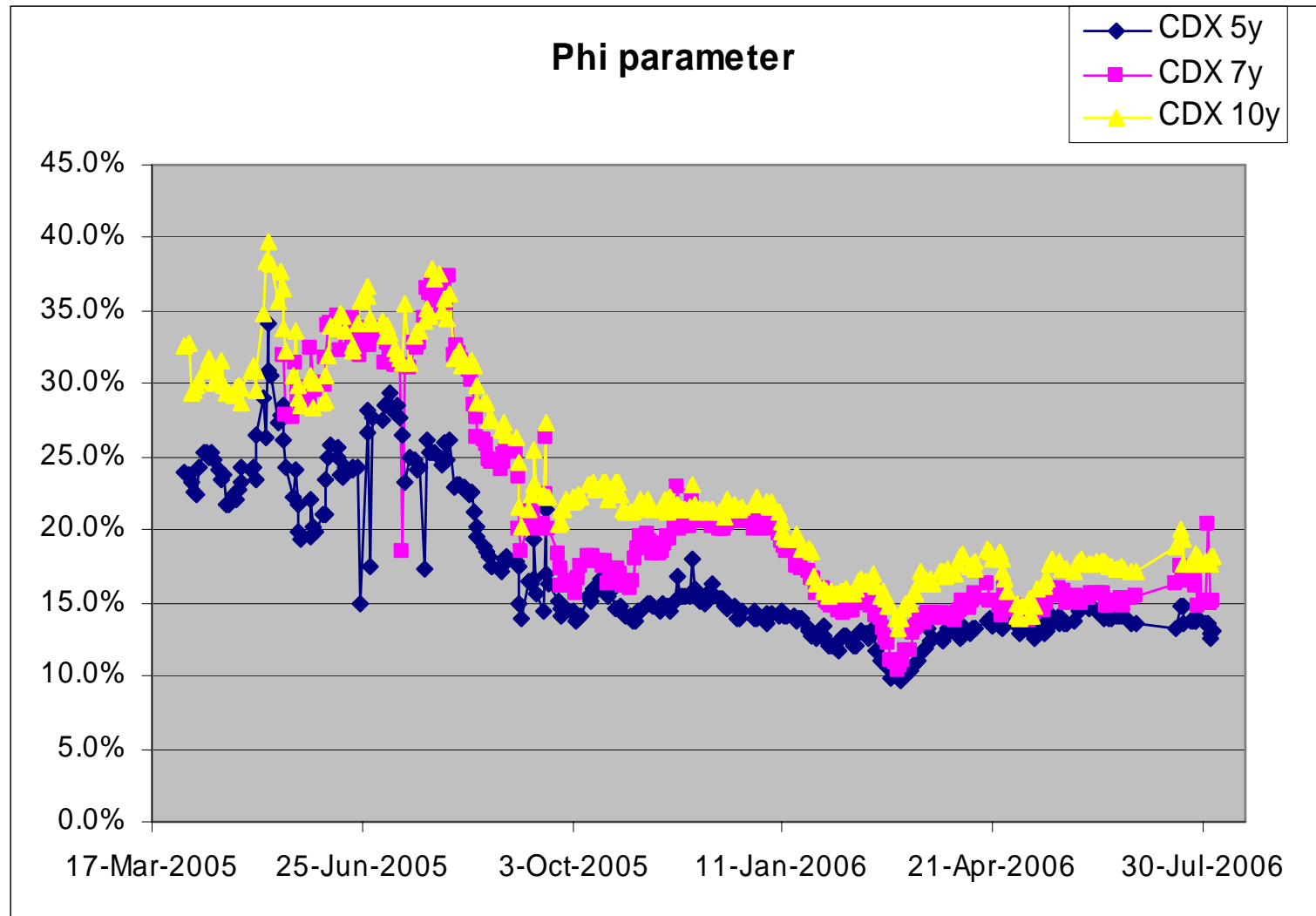
Maturity	iTraxx S6	
	Gamma	Phi
5y	29.0%	13.1%
7y	23.2%	15.3%
10y	12.1%	19.6%

- Also calibrates to CDX HY basket, Totem bespoke

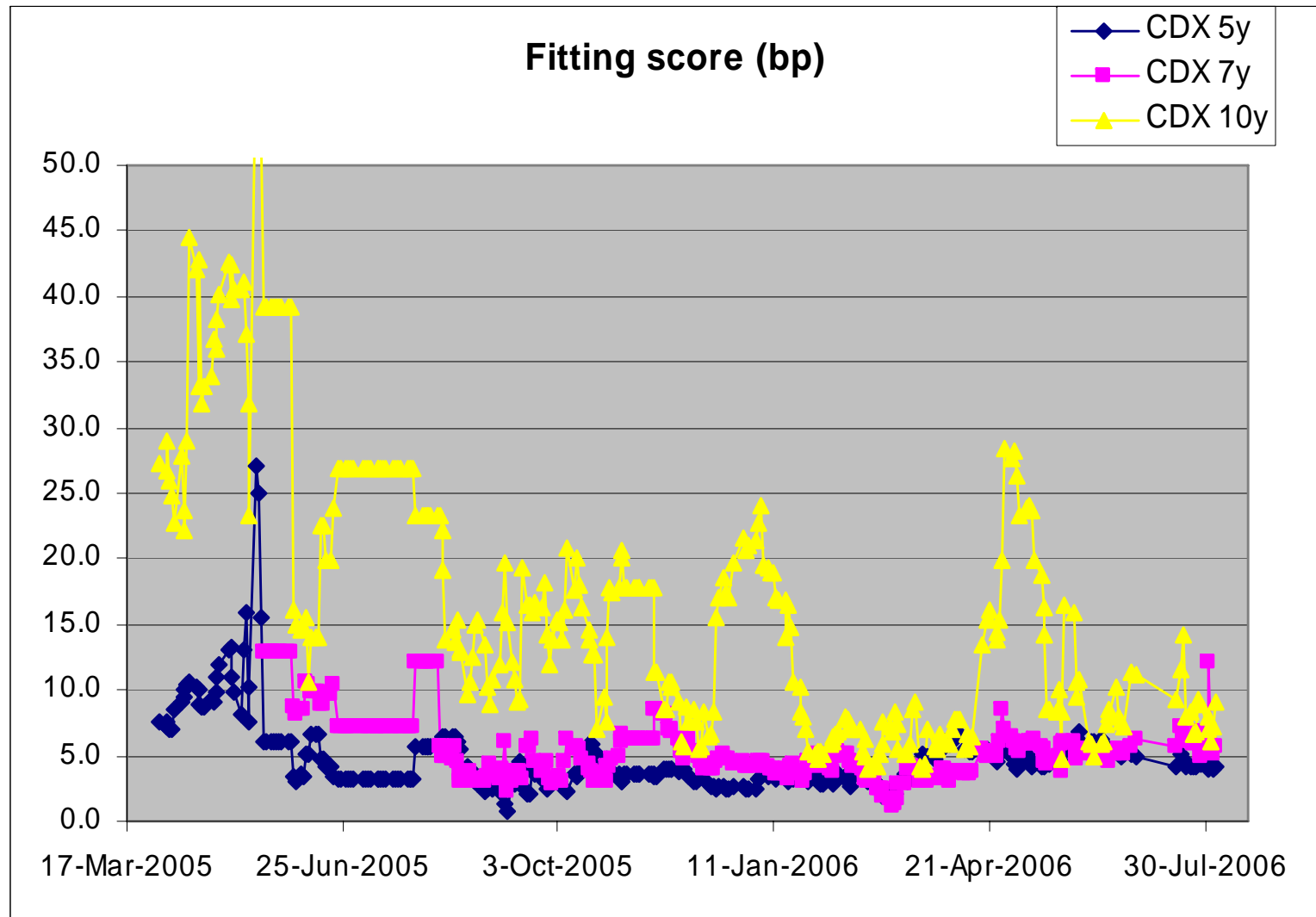
# Parameters (Gamma) – Gamma



# Parameters (Gamma) - Phi



# Parameters (Gamma) – Fitting score



# Hedging

- Hedge gamma and phi to flatten skew risk
- Parameter-based hedging rather than tranche-based
  - Similar in spirit to stochastic vol hedging in IR
- Can hedge different parts of the capital structure against each other
- Can price and hedge bespoke with liquid tranches
- Back tests well during Auto downgrade crisis May 2005
  - large gamma sell-off
  - some hedge funds had a “positive carry” trade (selling equity protection, buying mezz protection): long phi, long gamma.

# Model properties

- Two intuitive parameters controlling the average level and slope of the Base Corr curve
- Tractable to implement with easy calibration
- Prices bespoke baskets and tranchelets
- Exotic products, such as CDO<sup>2</sup>, long-short, etc
- Risks to both individual credits and skew parameters
- Hedging of skew params across tranches and baskets
- Relative-value indications
- Term-structure and path-dependency possible, but needs more work

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