# Levy process dynamic modelling of single-name credits and CDO tranches


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## 1. Introduction

The standard Gaussian copula model, with its overlay of base correlation, is useful but not ideal. It is essentially a static look-up table which does not model the dynamics of the process. It is hard to extend to bespoke baskets or other products, and it readily admits arbitrage.

Various models have been proposed to address these problems. These include both structural models which drive the value of the firm (or proxy) and reduced form models which drive default intensity rates or the loss distribution. To some extent, much academic research has favoured structural models while practitioners tend to use reduced form.

This paper is focused on practitioner requirements, but nonetheless presents a family of structural models. The model has been developed with attention given to the practitioner needs for both intuitive dynamics and for ease of technical implementation and calibration. It is a jump-based model which allows both global jumps and idiosyncratic jumps. This is important because some credit jump events are global (such as the GM downgrade in May 2005) and others are name-specific (such as Parmalat and Railtrack).

## 2. Model description

The economic ideas behind the new model are twofold. Firstly, the tails of distributions are important because default events are tail events, and Gaussian tails are too light to match the market, but jump processes introduce heavier tails. Secondly, jumps can happen for either market-wide or name-specific reasons.

Our new models are based on the gamma process $\Gamma(t;\gamma,\lambda)$ as a common building block, denoting the pure-jump increasing Levy process with intensity measure

$$\nu(x) = \frac{P(\Delta X_t \in dx)}{dx\,dt} = \gamma x^{-1} \exp(-\lambda x), \quad x > 0.$$

The parameter $\gamma$ controls the rate of jump arrivals and $\lambda$ is the inverse average jump size. Jumps of size [$x$, $x+dx$] occur as a Poisson process with intensity $\nu(x)\,dx$. Applebaum (2004) is a good reference for more details on Levy processes and their stochastic calculus.

The key idea for creating dependency between credits is the following lemma.


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***Lemma (Multivariate Levy process)***. For any Levy process $X(t)$, we can construct an $n$-dimensional multivariate Levy process with equal marginal distributions of $X(t)$ and correlation $\phi$. Take a global factor, $X_g$ and idiosyncratic factors $\tilde{X}_i$ (i=1,..,$n$) to be independent identically distributed copies of $X(t)$, and define the $i$th process to be the sum

$$X_i(t) = X_g(\phi t) + \tilde{X}_i\left((1-\phi)t\right).$$

Then each $X_i(t)$ has the same distribution as $X(t)$. The parameter $\phi$ is the proportion of the movement of an entity due to global events. If $X(t)$ has finite second moments, then $\phi$ is the correlation between $X_i(t)$ and $X_j(t)$.

The proof is trivial from the properties of a Levy process.

A first application gives our primary model, which we call the **Gamma model (G)**. We model that an entity's structural value under the risk-neutral measure is given by the exponential martingale

$$S_t^i = S_0^i \exp(X_t^i + \mu t)\text{, with}$$

$$X_t^i = -\Gamma_g(t;\phi\gamma,\lambda) - \Gamma_i(t;(1-\phi)\gamma,\lambda)\text{, and } \mu = \gamma \log(1+\lambda^{-1}),$$

where the two gamma processes $\Gamma_g$ and $\Gamma_i$ are independent global and idiosyncratic factors, with equal jump size but different intensities. The entity is deemed to have defaulted by time $t$ if $S_t^i \le k_t^i$, or equivalently if $X_t^i \le \theta_t^i$, where $k_t^i$ and/or $\theta_t^i$ are calibrated to match the entity's market default probabilities. This "european" default condition is a tractable approximation to the exact barrier condition $\inf\{S_s^i : s \le t\} \le k^i$.

The economic idea underlying the model is that the (tradable) value process is a positive martingale which has a general upward trend but suffers from down jumps as (bad) news arrives. Many of these jumps are small, but some may be quite large.

Various extensions to this basic model are possible. We shall consider three in particular.

**Variance Gamma model (VG)**. The Gamma model only has down jumps, so it might make sense to add up-jumps to reflect the arrival of good news (either global or systemic). This can be modelled as

$$X_t^i = -\Gamma_g^d(t;\phi\gamma,\lambda) + \Gamma_g^u(t;\phi\gamma,\lambda_u) - \Gamma_i^d(t;(1-\phi)\gamma,\lambda) + \Gamma_i^u(t;(1-\phi)\gamma,\lambda_u).$$

The new parameter $\lambda_u$ is the inverse jump-size of the up-jumps, but up and down jumps have equal intensity. This is equivalent to the model presented, using different notation, in Moosbrucker (2006). It itself is based on the variance gamma process of Madan et al (1998) which is the difference of two gamma processes

$$VG(t;\gamma,\lambda,\lambda_u) = \Gamma_u(t;\gamma,\lambda_u) - \Gamma_d(t;\gamma,\lambda),$$

and it can also be written as a gamma-time change of a drifting Brownian motion (see section 5 for details).

**Brownian Gamma model (BG)**. We can also add a continuous term, representing noise to complement the jumps. This can be written as

$$X_t^i = -\Gamma_g(t;\phi\gamma,\lambda) + \sqrt{\rho}W_t^g - \Gamma_i(t;(1-\phi)\gamma,\lambda) + \sqrt{1-\rho}W_t^i,$$

where $W^g$ and $W^i$ are global and idiosyncratic Brownian noise terms respectively. The continuous correlation $\rho$ may be different from the jump correlation $\phi$. We note that as the jump intensity $\gamma$ decreases to zero, this model reduces to the Gaussian copula.

**Catastrophe Gamma model (CG)**. In this variant, we add a low-intensity high-impact global "catastrophe" term. The idea behind this model is to create disaster scenarios which might help reprice senior tranches. The model's form is that of a Gamma model plus a Poisson catastrophe term

$$X_t^i = -\Gamma_g(t;\phi\gamma,\lambda) - \Gamma_i(t;(1-\phi)\gamma,\lambda) - Y.C_t\,,$$

where $C(t)$ is a very low-intensity global Poisson process, and $Y$ is an independent exponential random variable with a very large mean. In other words, the catastrophe is a Levy process with intensity measure $\nu(x) = \varepsilon K^{-1}\exp(-x/K)$, with small $\varepsilon$ and large $K$.

Combinations of these models are also possible, such as Brownian-Variance-Gamma (BVG), Catastrophe-Variance-Gamma (CVG), and so on. All the models share the common feature of gamma process down-jumps which contain both global and idiosyncratic jumps.

***Existing models***

There has been increasing interest in jump models recently, under both reduced-form and structural approaches, often using the variance gamma process. The original intuition of the VG model developed by Madan et al (1998) is that of a time-changed Brownian motion with drift. The time-change can be thought of as an "information time clock". Whilst this is a useful intuition for equity skew modelling, it is less helpful for portfolio credit models as it encourages a global "time-change clock" $A(t)$ which does not permit idiosyncratic jumps.

Using reduced form, Joshi and Stacey (May 2005) model the intensity as driven by a Levy process clock which is the sum of gamma processes, and they achieve CDO calibration. An economic drawback is that it does not allow idiosyncratic spread jumps and its practical limitation is that it requires Monte Carlo. Schoutens (Jan 2006) drives the intensity as a gamma OU process with spread up-jumps, and has success with CDS calibration but not CDO calibration.

As a structural model, Luciano and Schoutens (Dec 2005) have a multivariate variance gamma process achieved by a global time-change clock. Again this has no idiosyncratic jumps and does not claim to match CDO prices. We will test a basic version of this model (naming it "Global Info Time") with

$$X_t^i = W_i(A_t^g) + \mu A_t^g\,,$$

where $W(i)$ is idiosyncratic Brownian motion and $A(g)$ is a global information time clock.

## 3. Single-name credits

One way of validating the economics of the Gamma model is to try to fit it to single names. In this case, the correlation phi is redundant, and the relevant parameters are gamma and lambda. The form of the model we use is $S_t^i = S_0^i \exp(X_t^i + \mu t)$, because the $S$-space threshold $k$ should be approximately constant in time, whereas the $X$-space threshold $\theta$ is not.

We want to attempt a simultaneous calibration of many single-name credits. Suppose we take a basket of credits, such as CDX 125 S6. There are two model parameters, gamma and lambda, which we assume are constant over all the names, plus a separate barrier threshold $\theta(i)$ for each name. For any $(\gamma, \lambda)$ pair and for each name in the basket, we take the market 5y

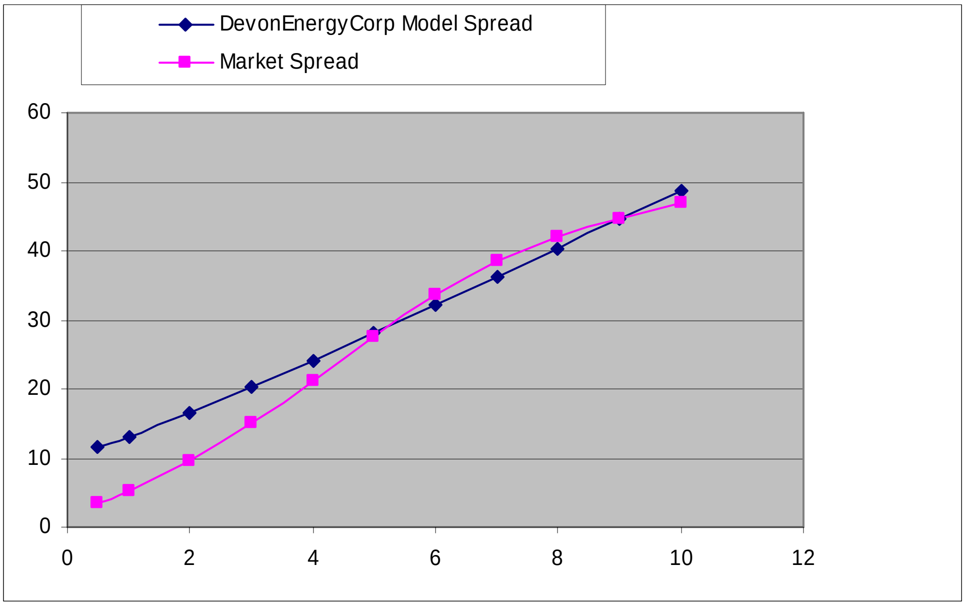
CDS spread for that name and perform a one-dimensional rootfind to determine the θ(*i*) value which matches the 5y market price. Using that θ(*i*), the model will then infer the rest of the CDS curve which can be compared with the market curve.

Our objective function is the average absolute spread errors both over the maturities 2y, 3y, (5y), 7y, 10y and over all the curves. We optimise this objective over the two-dimensional (γ, λ)-space.

As it happens, there is a range of points having relatively good fits. The table shows the average spread error for the CDX 125 S6 basket as at 19 April 2006 (gamma is the left-hand column; lambda is the top row), with values less than 7bp highlighted.

| | 20% | 40% | 60% | 80% | 100% |
|---|---|---|---|---|---|
| 5% | 16.2 | 17.4 | 18.0 | 18.5 | 18.8 |
| 10% | 12.2 | 14.0 | 15.2 | 15.9 | 16.5 |
| 20% | 6.3 | 8.4 | 10.1 | 11.4 | 12.3 |
| 40% | 8.6 | 5.6 | 5.0 | 5.6 | 6.4 |
| 60% | 16.6 | 9.7 | 6.5 | 5.0 | 4.6 |
| 80% | 25.1 | 15.4 | 10.3 | 7.4 | 5.7 |
| 100% | 33.0 | 21.1 | 14.6 | 10.6 | 8.0 |

Taking gamma to be 20% and lambda to be 20%, the basket's median fit error is 6.9bp (Devon Energy Corp). Its market and model CDS curve is shown in the graph below.




The fit does not have to be perfect, since there are only two parameters for 125 separate curve shapes, but it does suggest that our economic model bears an approximate resemblance to reality.

## 4. Baskets

We are now ready to use our models to price tranches on baskets. Let us start with the basic Gamma model. It is sufficient to model the log-value process $X(t)$ which is made up of a global gamma process and an idiosyncratic gamma process

$$X_t^i = -\Gamma_g(t;\phi\gamma,\lambda) - \Gamma_i(t;(1-\phi)\gamma,\lambda).$$

The default thresholds $\theta_t^i$ are calibrated to match the market survival probabilities of entity *i* at the swap times *t*. Since lambda is just a scale factor for *X*, and thus for θ, it can be ignored.

The calibration process for a given basket and a particular maturity is as follows. We start by fixing lambda at any arbitrary rate (for example 100%). For any gamma-phi pair we can calibrate the thresholds $\theta_t^i$ for every name *i* and time *t*. The model is now parameterised and can be run to give the spreads of tranches within the basket's capital structure. A goodness of fit score can be calculated by comparing the model's tranche prices with the market's. We choose our objective function to be the root mean squared error

$$V(\gamma,\phi) = \left( \tfrac{1}{m} \sum_{\substack{tranche\\ j=1}}^{m} \left( S_j^{\text{market}} - S_j^{\text{model}} \right)^2 \right)^{1/2} .$$

We then vary (γ, ϕ) to find the minimum value of *V*. Details of the implementation can be found below. The results of calibrating to CDX 125 S6 and iTraxx 125 S5 on 19 April 2006 are shown in the tables

| | **5y CDX** | | **7y CDX** | | **10y CDX** | |
|---|---|---|---|---|---|---|
| Tranche | Market | Model | Market | Model | Market | Model |
| 0% - 3% | 1,287.4 | 1,288.8 | 1,642.1 | 1,643.7 | 1,752.3 | 1,766.5 |
| 3% - 7% | 83.9 | 81.9 | 217.3 | 215.3 | 549.7 | 532.6 |
| 7% - 10% | 16.6 | 26.5 | 34.6 | 37.8 | 92.6 | 95.5 |
| 10% - 15% | 8.5 | 16.0 | 18.1 | 23.9 | 44.9 | 42.9 |
| 15% - 30% | 5.6 | 7.8 | 7.1 | 13.4 | 14.6 | 23.1 |
| 30% - 100% | 2.9 | 1.3 | 5.2 | 2.7 | 7.0 | 4.8 |
| Best fit score (bp) | | **5.3** | | **4.0** | | **9.9** |

| | **5y iTraxx** | | **7y iTraxx** | | **10y iTraxx** | |
|---|---|---|---|---|---|---|
| Tranche | Market | Model | Market | Model | Market | Model |
| 0% - 3% | 1,022.6 | 1,023.5 | 1,366.1 | 1,366.8 | 1,512.9 | 1,556.3 |
| 3% - 6% | 59.0 | 57.3 | 172.9 | 172.5 | 527.7 | 471.6 |
| 6% - 9% | 15.7 | 21.0 | 41.4 | 40.8 | 96.9 | 121.1 |
| 9% - 12% | 8.3 | 13.1 | 21.0 | 24.9 | 44.6 | 57.5 |
| 12% - 22% | 3.7 | 7.3 | 7.7 | 14.2 | 16.7 | 27.3 |
| 22% - 100% | 2.1 | 1.2 | 4.1 | 2.7 | 5.8 | 3.6 |
| Best fit score (bp) | | **3.4** | | **3.2** | | **31.3** |

The fitting is generally quite good. The equity and junior mezzanine tranches generally fit very well, though iTraxx 10y is less good. The main error is between the senior tranche, which the model has too high, and the super-senior tranche, which the model has too low.

The parameters which achieved this fit are

| | **CDX** | | **iTraxx** | |
|---|---|---|---|---|
| Maturity | Gamma | Phi | Gamma | Phi |
| 5y | 18.2% | 13.3% | 19.4% | 11.7% |
| 7y | 7.2% | 15.0% | 10.9% | 14.7% |
| 10y | 4.9% | 18.5% | 14.8% | 16.2% |

We can repeat this fitting process for the other models described and for a variety of dates (weekly from 12 October 2005 to 5 April 2006), taking the average over the dates for each basket/maturity combination. The results of the fits are in the table.

| Model | CDX 5y | CDX 7y | CDX 10y | iTraxx 5y | iTraxx 7y | iTraxx 10y | **Average (bp)** |
|---|---|---|---|---|---|---|---|
| Cat Gamma (CG) | 1.4 | 7.9 | 15.4 | 1.1 | 7.0 | 8.7 | 6.9 |
| Variance Gamma (VG) | 2.9 | 9.6 | 15.7 | 2.9 | 9.6 | 7.0 | 8.0 |
| **Gamma** | **3.3** | **7.7** | **17.2** | **3.2** | **6.8** | **17.0** | **9.2** |
| Brownian Gamma (BG) | 4.7 | 11.1 | 18.3 | 3.9 | 9.2 | 13.8 | 10.2 |
| Brownian VG | 2.8 | 21.9 | 44.2 | 2.3 | 18.2 | 40.6 | 21.7 |
| Cat VG | 1.4 | 28.6 | 48.1 | 1.0 | 26.4 | 34.7 | 23.4 |
| Global Info Time | 77.0 | 28.5 | 66.1 | 38.0 | 32.3 | 49.5 | 48.6 |
| Gaussian copula | 38.9 | 66.1 | 76.3 | 33.6 | 75.7 | 83.9 | 62.4 |

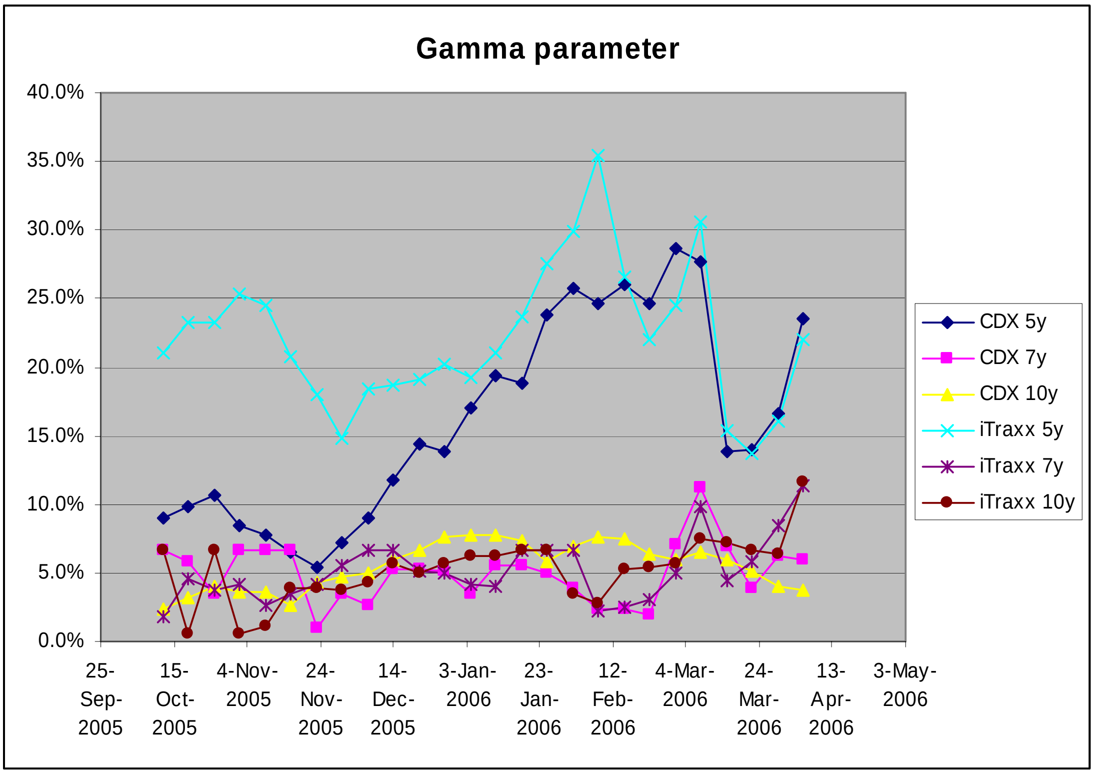
This table contains some interesting results. The Gamma model and its three basic variants (CG, VG, and BG) are all doing well with average errors between 7bp and 10bp. The BVG and CVG models, which have symmetric up and down jumps, have errors between 20bp and 25bp, and are not quite as good. The "Global Info Time" model, as expected, is not a very good fit. The Gaussian copula at 62bp is included for reference.

For the 5y maturity, the Catastrophe Gamma (CG) model fits particularly well.

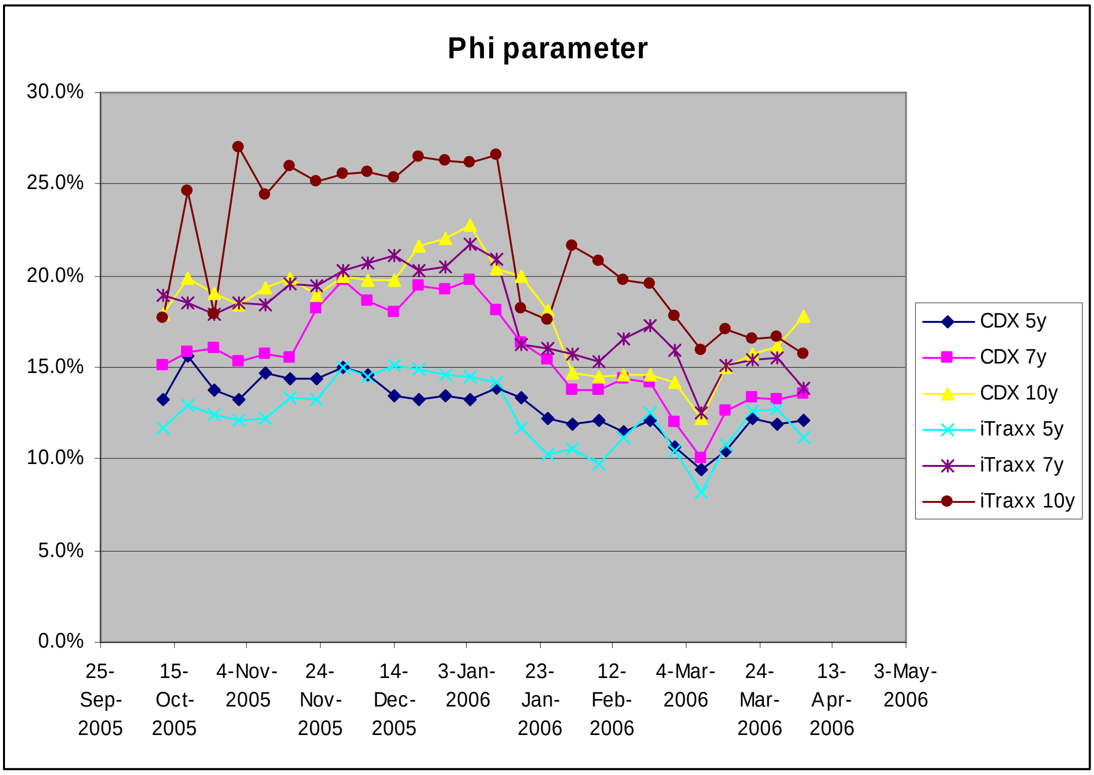
For the 7y and 10y maturities, many models often reduce to the Gamma model. Each of the CG model (68% of fits), VG model (47% of fits), and BG model (100% of fits) have fitted parameters which (almost) reduce the model back to the simple Gamma.

The VG and BG models cannot exactly reduce to the Gamma model because lambda values would have to go to zero, but the optimiser keeps them bounded away from zero for numerical tractability. So paradoxically the Gamma model sometimes outperforms VG and BG slightly.

The symmetric up and down jumps of BVG and CVG do not seem to help fitting particularly.



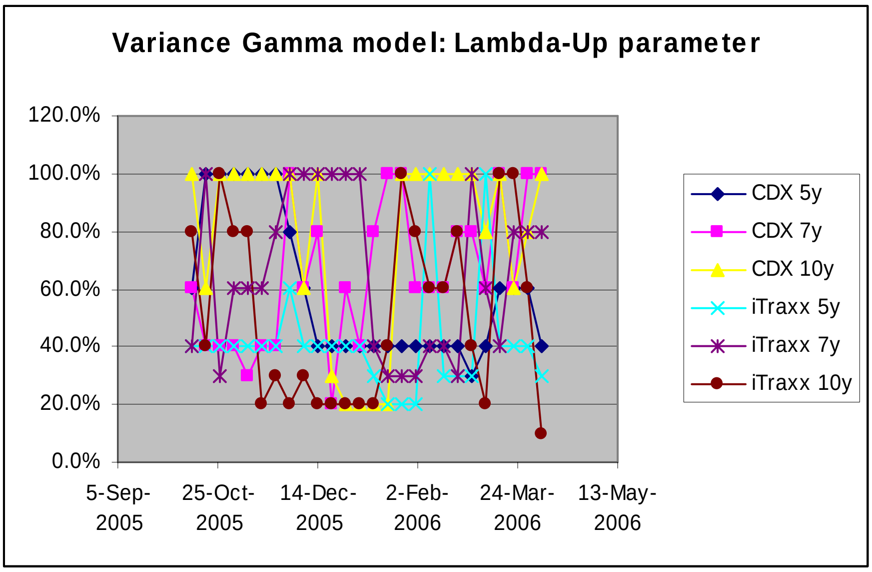
Parameter stability is also an important practical concern. If the fitted parameters vary widely from day to day, the model will be less useful for hedging and will be less credible as a description of the process dynamics. The charts show the fitted gamma and phi parameters through time and for the various baskets and maturities.




We see that the parameters are mostly stable and that CDX and iTraxx parameters are similar. The main movement is that 5y gamma increased during the winter of 2005/6 and again around the index roll in March 2006. Higher gamma corresponds to spreads reducing in the equity and senior tranches, but increasing in the mezzanine tranches. This agrees with the movements in the market at that time. The CDX parameters are similar to the iTraxx parameters, which could allow bespoke baskets to be priced with a blended mixture of those parameters.

| Model | Gamma | Phi | Lambda | CatRate/Rho |
|---|---|---|---|---|
| Gamma | 38% | 15% | | |
| Variance Gamma (VG) | 36% | 16% | 44% | |
| Cat Gamma (CG) | 81% | 22% | | 149% |
| Brownian Gamma (BG) | 43% | 17% | 0% | 218% |
| Brownian VG | 48% | 19% | 75% | 209% |
| Cat VG | 38% | 37% | | 35% |

Parameter stability can be compared across models. The stability table above shows a measure of stability for each parameter under each model. The measure used is the standard deviation of the fitted parameter values divided by the average of the fitted parameter values. The Gamma model is relatively stable. The Variance Gamma model is apparently stable, but the stability measure disguises the problems with lambda-calibration which are visible in the picture below. The CG, BG, and BVG models all have some unstable parameters. Further the CVG model has a specious stability for 7y and 10y because it was unable to calibrate well.




## 5. Implementation

The models presented here can be implemented in a similar way to many existing Gaussian copula implementations. In particular, they do not require Monte Carlo simulation, though it is possible.

Let us formulate the Gaussian copula model in a similar way to our Levy process dynamics as

$$X_i(t) = W_g(\rho t) + W_i\big((1-\rho)t\big),$$

where $W(g)$ and $W(i)$ are global and idiosyncratic Brownian motions. Thus $X_i$ is also a Brownian motion. Let us write $F(x;t)$ for the common distribution function of $X(t)$, $W_g(t)$, and $W_i(t)$. A sketch of the implementation could run as follows:

(1) For each time $t$, calculate the threshold $\theta_i(t) = F^{-1}(p_i(t);t)$, where $p_i(t)$ is the default probability of entity $i$ by time $t$.

(2) Integrate over the possible values of the global factor $W(g)$, which has distribution function $F(x;\rho t)$. We can use either simple methods such as Simpson integration, or more sophisticated quadrature techniques. Both methods use an approximation of the form

$$E(payoff(W_g)) \cong \sum_{k=1}^{m} \alpha_k E(payoff \mid W_g = y_k),$$

where $y(k)$ are a discrete set of values of $W(g)$, and $\alpha(k)$ are some weights.

(3) Given that the global factor ($W(g) = y$), we calculate the conditional default probabilities of each entity

$$p_i(y,t) = P\big(X_i(t) \le \theta_i(t) \mid W_g = y\big) = F\big(\theta_i(t) - y;(1-\rho)t\big).$$

(4) With these conditional default probabilities, we calculate the conditional expectation of the payoff. This is helped by the conditional independence of the entities' values given $W(g)$. The expectation can be performed by approximations such as a normal-approximation to the basket loss, or the ingenious bucket algorithm of Hull and White (2004).

To change from the Gaussian copula to our new Levy models, all we have to do is replace the distribution function *F* used in steps (1), (2), and (3). Step (4) is unaltered. So the problem reduces to calculating the marginal distribution function for the various models we have used.

### *Calculating the distribution function*

**Gamma model**. The Gamma model has marginal gamma distributions. Their distribution is already well approximated. See, for example, section 6.2 of *Numerical Recipes* (Press et al, 1988). Quadrature integration against a gamma random variable is also possible, as implemented in routine `gaulag` of *Numerical Recipes* section 4.5. Run-time performance for the Gaussian copula and the Gamma model should also be broadly similar.

For calculating the inverse of the distribution function, it is effective to perform interval bisection to bracket the root initially, since the distribution function is monotonic, and then apply some Newton-Raphson iterations to polish it.

**Variance Gamma model**. There is a time-change representation of the VG process as

$$VG(t;\gamma,\lambda,\lambda_u) = W(A_t) + \mu A_t \,,$$

where $W(t)$ is a Brownian motion, $A(t)$ is a gamma $\Gamma(t;\gamma,\frac{1}{2}\lambda\lambda_u)$ process, and $\mu$ is the drift $\frac{1}{2}(\lambda_d - \lambda_u)$. Thus $P\big(VG(t;\gamma,\lambda,\lambda_u) \le x\big) = P\big(Z\sqrt{A_t} + \mu A_t \le x\big)$. The probability can then be expressed as an integral conditional on the value of $\Gamma = A(t)$ as

$$P\big(Z\sqrt{\Gamma} + \mu\Gamma \le x\big) = \int_0^\infty f_\Gamma(y)\Phi\left(\frac{x-\mu y}{\sqrt{y}}\right)dy \,.$$

This integral can be performed efficiently using the gamma quadrature integration mentioned above. Numerical difficulties may occur when the ratio $\lambda_u / \lambda$ is extreme (larger than 10), so these cases might be excluded.

Other models, such as Brownian Gamma, can be handled in similar ways.

### *Performing the optimization*

**Gamma model**. The optimization for the Gamma model is relatively straightforward. There are only two parameters (gamma and phi) and they both have non-trivial and different effects on the tranche spreads. We use an optimiser similar to the Levenberg-Marquardt method, described in section 15.5 of *Numerical Recipes*. About half-a-dozen iterations are enough to get a good calibration.

**Other models**. We use the same optimizer, but the situation is more complicated. The function mapping parameters to tranche spreads is significantly non-linear and the presence of semi-redundant parameters increases the difficulty. For such difficult parameters, we try optimizing whilst keeping that parameter fixed, and then we vary the parameter and optimize again. This is slow but effective. The striated levels shown in the VG lambda-up calibration graph above are an example of this procedure.

## 6. Summary and conclusions

We have introduced a family of Levy process structural models, along with a simple correlation structure. The family is centred around the Gamma model.

All the models fit the market CDO prices better than the Gaussian copula. They also describe the dynamics of the correlated processes in a consistent arbitrage-free way. The models are capable of being extended to more complex products, though this has not been attempted.

It is, of course, possible to develop more sophisticated models within this framework to achieve better accuracy or more precise dynamics. The question of exact calibration of the senior and super-senior still remains open.

Of the individual models considered, the simple Gamma model has a number of advantages. It fits relatively well, and nearly as well as the more complicated models. Its parameters are the most stable of the feasible models. Its implementation and calibration is tractable, and somewhat easier than for others. The Gamma model seems, on this evidence, the best initial model for practical use.

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