Fixed Income Division

Dynamic modelling of single-name credits and CDO tranches

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Outline of talk

- Levy processes
- Correlated Levy processes
- Credit modelling
- Single-names
- CDO portfolios
- Parameters

Motivation and intuition

- Credit is about extreme events
- Extreme events through jumps
- Jumps can be global or idiosyncratic
- Need a dynamic model, arbitrage-free
- Tractable

Levy processes

- *Poisson process* – jumps of size 1 occur at rate λ
- *Levy process* – jumps of size *x* occur at rate $\nu(x)dx$
- Formally $X(t)$ is a Levy process if it has stationary independent increments, that is
  - $X(s+t) - X(s)$ is independent of $(X(u)$: $u<=s)$, and is distributed as $X(t)$.
- All Levy processes are made up of drift, Brownian motion and jumps

$$E(\exp(\theta X_t)) = \exp(t\psi(\theta))$$

$$\psi(\theta) = \mu\theta + \tfrac{1}{2}\sigma^2\theta^2 + \int (e^{\theta x} - 1)\nu(x)dx$$

- Drift μ, Brownian volatility σ, jump measure ν.

Gamma process

- *Gamma process* – pure jump increasing process with jump measure

$$v(x) = \gamma x^{-1} \exp(-\lambda x)$$

- Marginal distribution is the continuous gamma distribution.
- Parameter γ is jump intensity
- Parameter λ is inverse jump size
- Notation $X(t) = \Gamma(t; \gamma, \lambda)$

- *Variance Gamma process* (Moosbrucker) – difference of two Gammas

$$X(t) = VG(t; \gamma, \lambda, \lambda_u) = \Gamma(t; \gamma, \lambda_u) - \Gamma(t; \gamma, \lambda)$$

Correlated Levy processes

- Start with any Levy process distribution $X(t)$.
- Make independent copies X_g (global factor) and $\tilde{X}_1, \ldots, \tilde{X}_n$ (idiosyncratic factors)
- Then define correlated Levy processes as

$$X_i(t) = X_g(\phi t) + \tilde{X}_i((1-\phi)t)$$

- So that each $X(i)$ has the same distribution as X, and the correlation between $X(i)$ and $X(j)$ is ϕ.

Linking back to Gaussian copula

- The Gaussian copula can be thought of as

$$X_i = \sqrt{\rho} Z_g + \sqrt{1-\rho} Z_i$$

- This can be made dynamic as

$$X_i(t) = \sqrt{\rho} W_g(t) + \sqrt{1-\rho} W_i(t)$$

- And then by the space-time equivalence of BM

$$X_i(t) = W_g(\rho t) + W_i((1-\rho)t)$$

- This now matches our general form.

Credit modelling – basic Gamma model

- Structural model with value of the firm *S*(*t*)

$$S_t = S_0 \exp(-\Gamma(t;\gamma,\lambda) + \mu t), \quad \mu = \gamma \log(1+\lambda^{-1})$$

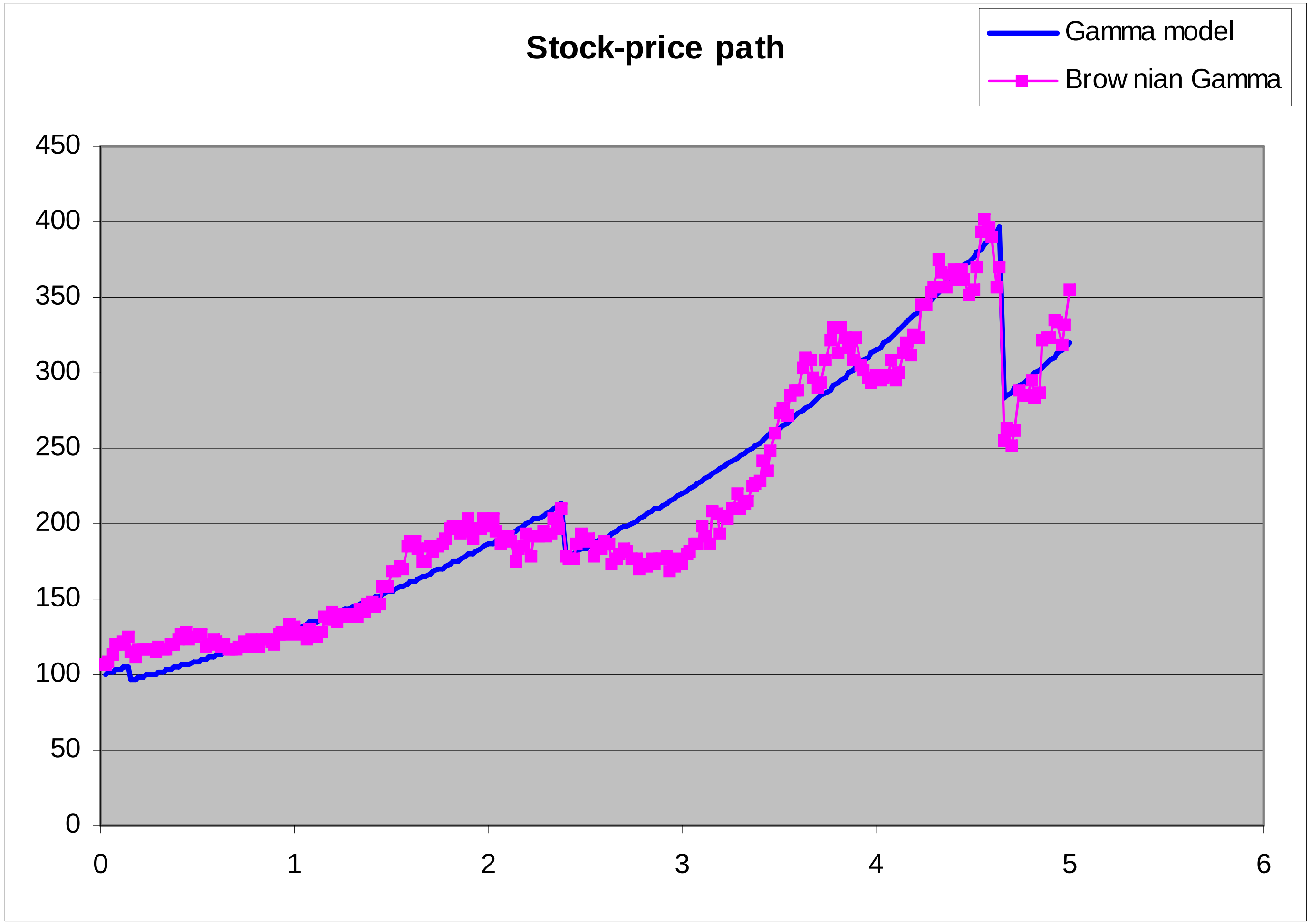
- Assume *S*(*t*) is a positive martingale with up-drift and down-jumps
- Entity defaults when *S*(*t*) goes below a threshold
- Correlation structure as before
- Reduces to log-normal as γ goes to infinity
- “No news is good news”

Credit modelling – extra features

- Add “good news” jumps with Variance Gamma instead of Gamma (VG).
- Add “catastrophe” term with extra low-intensity high-impact global factor (CG).
- Add Brownian term to get continuous random movement as well (BG).

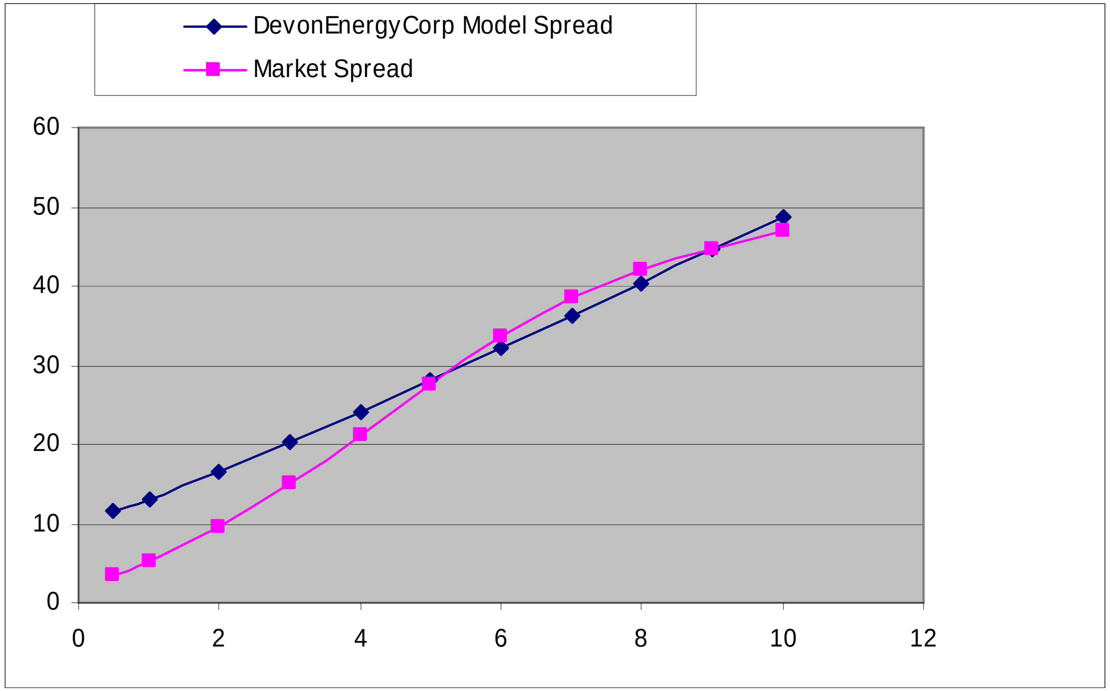
- Also various combinations, such as Brownian-Variance-Gamma (BVG), Catastrophe-Variance-Gamma (CVG), etc.

Credit modelling – sample path



Single-name credit dynamics for Gamma

- Can fit (γ, λ) to basket of credits, such as CDX 125 S6.
- Average spread error is only 6bp (at 19 April 2006)
- Median case, Devon Energy Corp, is shown



CDO Pricing

- We can reformulate the model as

$$X_t^i = -\Gamma_g(t;\phi\gamma,\lambda) - \Gamma_i(t;(1-\phi)\gamma,\lambda)$$

- Decomposes the log-value into global and idiosyncratic Gamma terms
- Default by *t*, if $X^i(t)$ below a threshold (European approximation)
- Lambda parameter is redundant due to scaling
- Phi parameter moves spread from junior to senior
- Gamma parameter moves spread from equity/senior to mezzanine

Fitting to CDO prices (CDX)

	5y CDX		7y CDX		10y CDX	
Tranche	Market	Model	Market	Model	Market	Model
0% - 3%	1,333.8	1,334.5	1,679.0	1,680.3	1,819.2	1,834.6
3% - 7%	96.5	96.3	240.0	238.7	575.0	559.4
7% - 10%	19.5	25.3	44.5	47.6	114.0	121.4
10% - 15%	10.0	12.2	19.5	26.7	51.5	50.2
15% - 30%	5.0	4.1	7.0	12.6	15.5	19.3
30% - 100%	3.1	0.3	4.0	2.0	5.3	2.4
Best fit score (bp)		**2.8**		**4.1**		**9.7**

Fitting as at 2 June 2006.

	CDX	
Maturity	Gamma	Phi
5y	56.0%	9.3%
7y	13.9%	13.8%
10y	15.4%	13.6%

Fitting to CDO prices (iTraxx)

	5y iTraxx		7y iTraxx		10y iTraxx	
Tranche	Market	Model	Market	Model	Market	Model
0% - 3%	1,075.6	1,076.0	1,391.7	1,392.1	1,486.6	1,491.5
3% - 6%	69.5	69.3	185.5	185.0	515.0	405.1
6% - 9%	19.0	21.1	46.0	47.9	119.0	127.0
9% - 12%	8.7	11.0	25.0	26.1	54.0	67.3
12% - 22%	3.8	4.7	8.3	12.5	20.5	32.0
22% - 100%	1.7	0.4	3.3	1.5	4.7	3.9
Best fit score (bp)		**1.4**		**2.1**		**45.6**

Fitting as at 2 June 2006.

	iTraxx	
Maturity	Gamma	Phi
5y	57.7%	8.5%
7y	24.3%	12.2%
10y	19.9%	19.6%

Model comparison

Model	CDX 5y	CDX 7y	CDX 10y	iTraxx 5y	iTraxx 7y	iTraxx 10y	**Average (bp)**
Cat Gamma	1.4	7.9	15.4	1.1	7.0	8.7	6.9
Variance Gamma	2.9	9.6	15.7	2.9	9.6	7.0	8.0
Gamma	**3.3**	**7.7**	**17.2**	**3.2**	**6.8**	**17.0**	**9.2**
Brownian Gamma	4.7	11.1	18.3	3.9	9.2	13.8	10.2
Brownian VG	2.8	21.9	44.2	2.3	18.2	40.6	21.7
Cat VG	1.4	28.6	48.1	1.0	26.4	34.7	23.4
Global Info Time	77.0	28.5	66.1	38.0	32.3	49.5	48.6
Gaussian copula	38.9	66.1	76.3	33.6	75.7	83.9	62.4

Both Gamma and its simple variants (CG, VG, and BG) do well.

BVG and CVG have symmetric up and down jumps which are not as good

Data weekly observed from 12 October 2005 to 5 April 2006.

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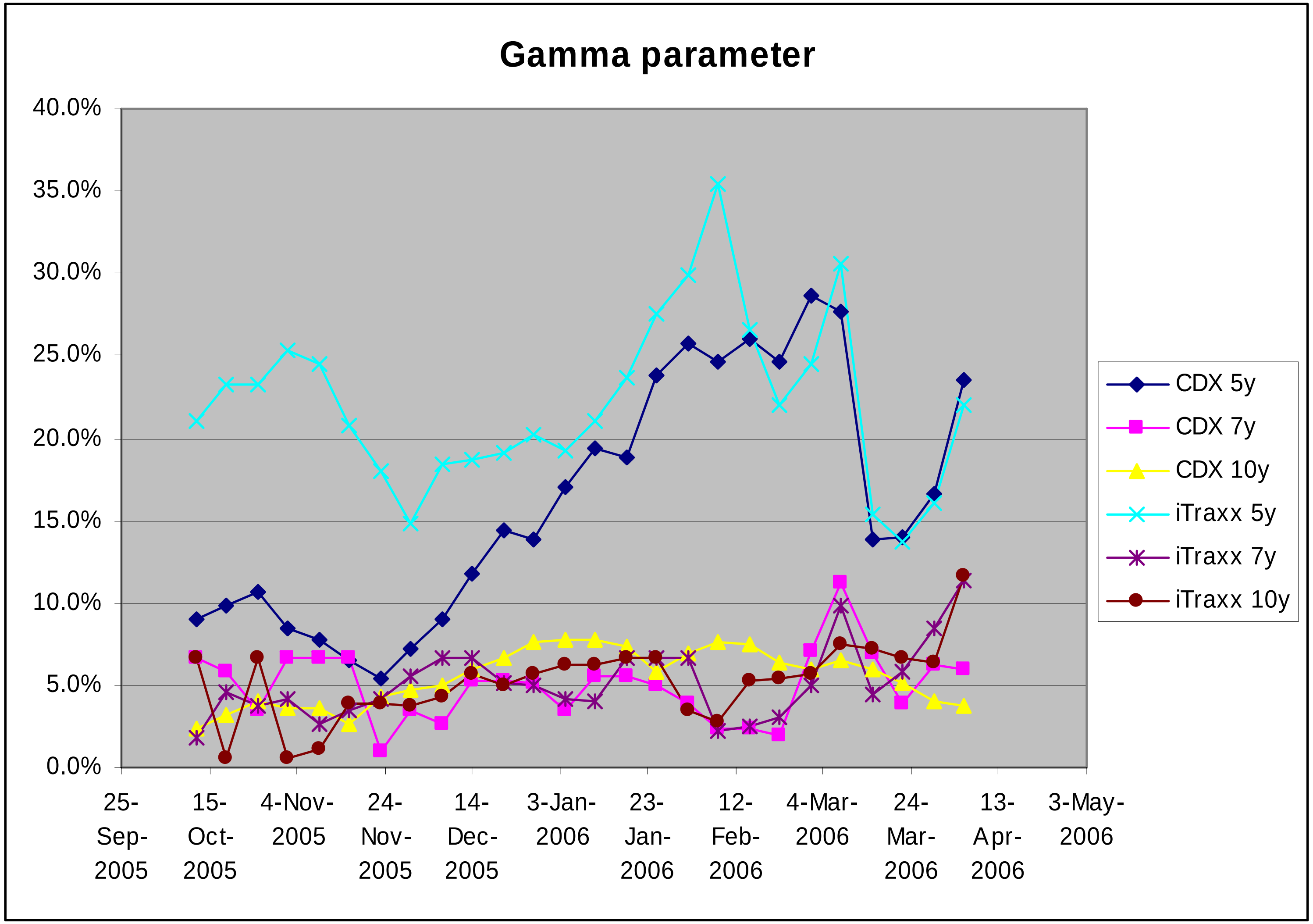
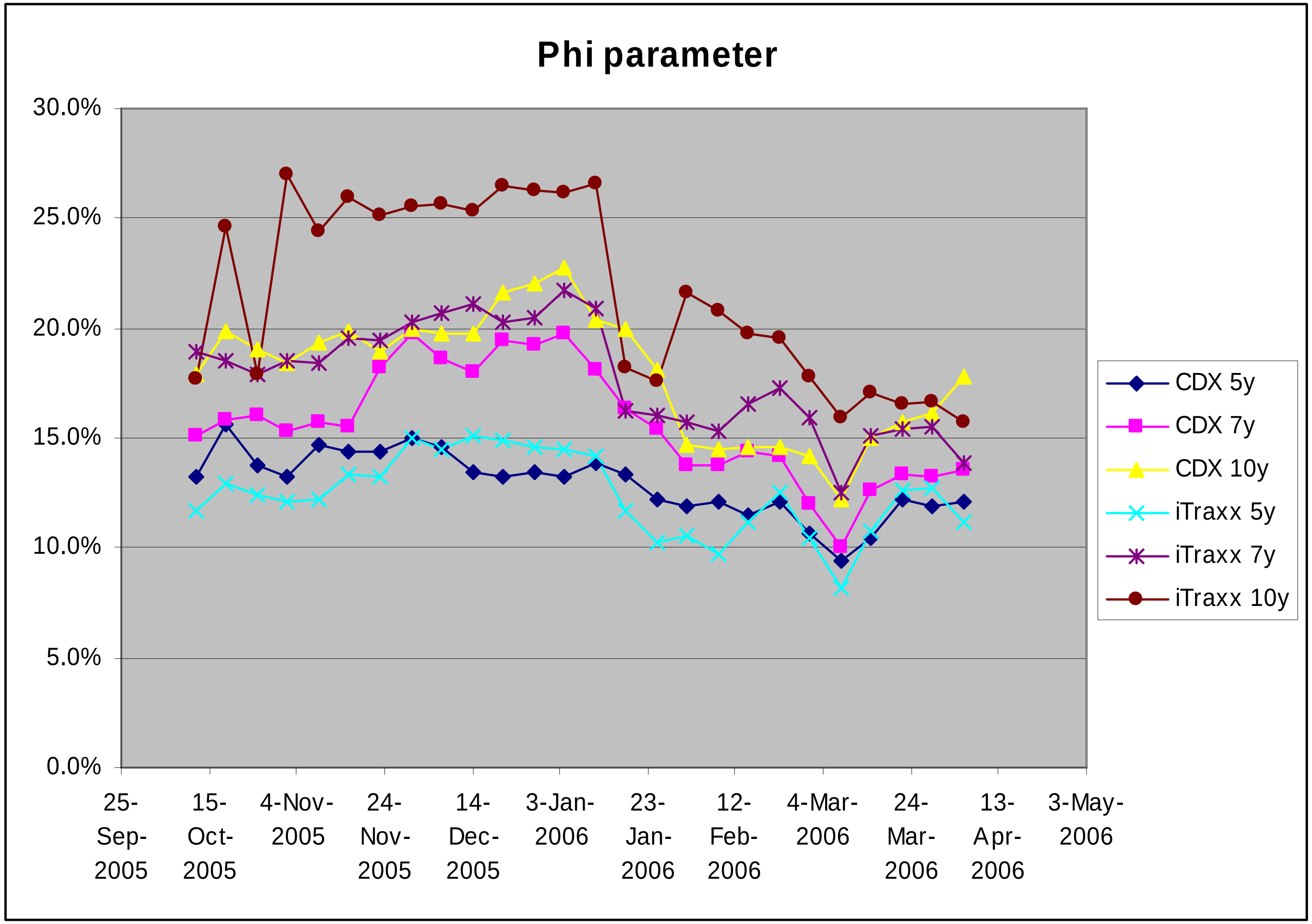
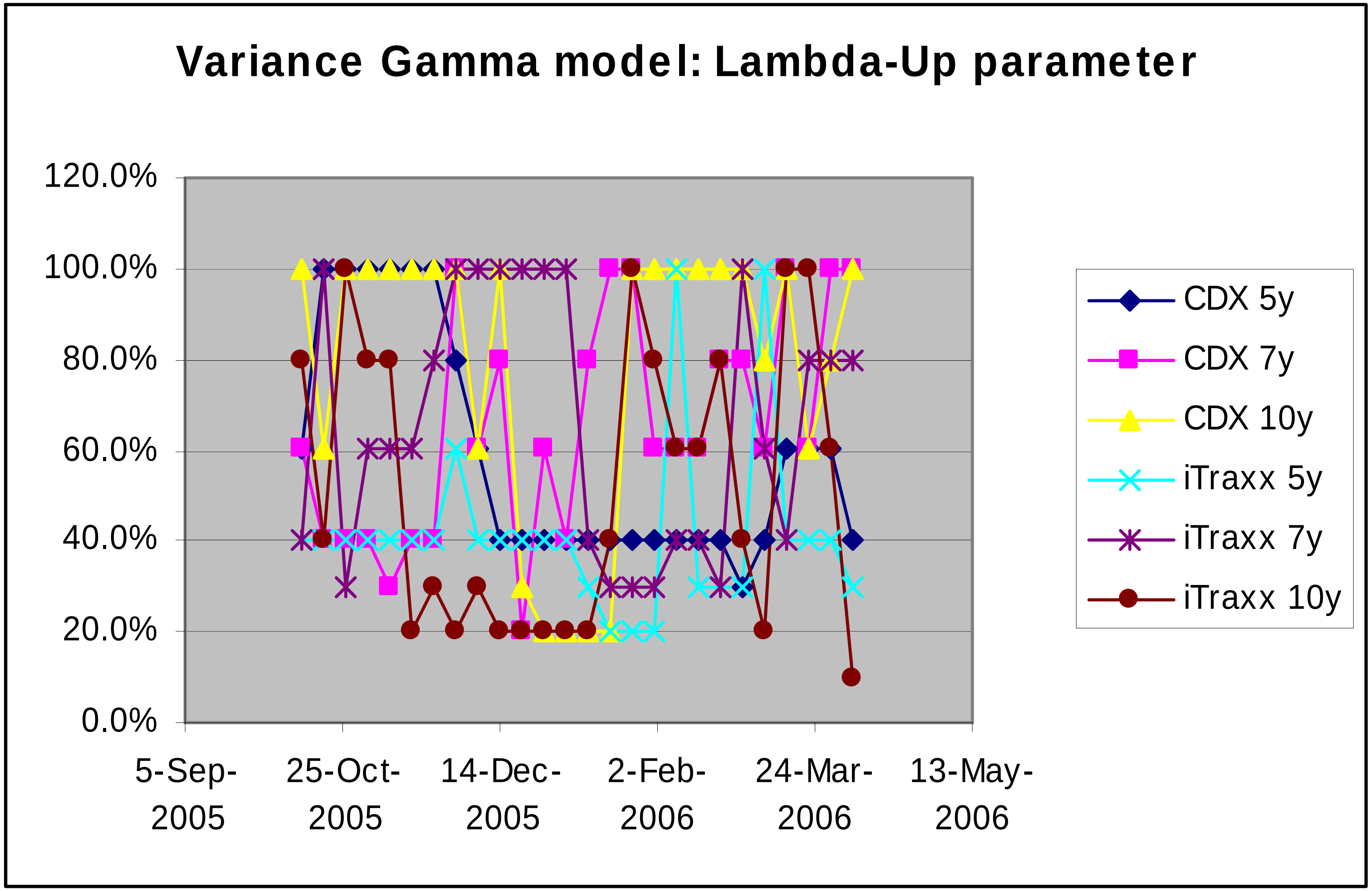
Parameters (G) – Gamma



Parameters (G) - Phi



Parameters (VG) – Lambda Up



Summary

- Levy processes create a family of models for credit
  - Jumps to match market tails
  - Simple correlation structure
- Single name dynamics consistent with CDS market
- Multivariate dynamics close to market CDO prices
- Simple Gamma model has good properties
  - Intuitive model
  - Acceptable fit
  - Stable parameters, not too many
  - Tractable to implement

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