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Inter-temporal risk parity

Nomura Global Quantitative Equity Conference 8th May 2014

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Inter-temporal risk parity strategy

- What is inter-temporal risk parity?
  - Systematic strategy rebalancing between a risky asset and cash
  - Weight of risky asset is chosen so that ex-ante risk is kept constant

$$r^{IRP}{}_t = r_t \frac{\kappa}{\sigma_t} + r_c \left(1 - \frac{\kappa}{\sigma_t}\right)$$

$r^{IRP}{}_t$	performance of the Inter-temporal Risk Parity strategy	σ_t	ex-ante volatility at t -1
r_t	performance of the risky asset	κ	pre-defined target risk budget
r_c	performance of cash	κ / σ_t	weight of risky asset

- Other names: *constant risk*, *inverse volatility weighting* and *iso-vol* (France).

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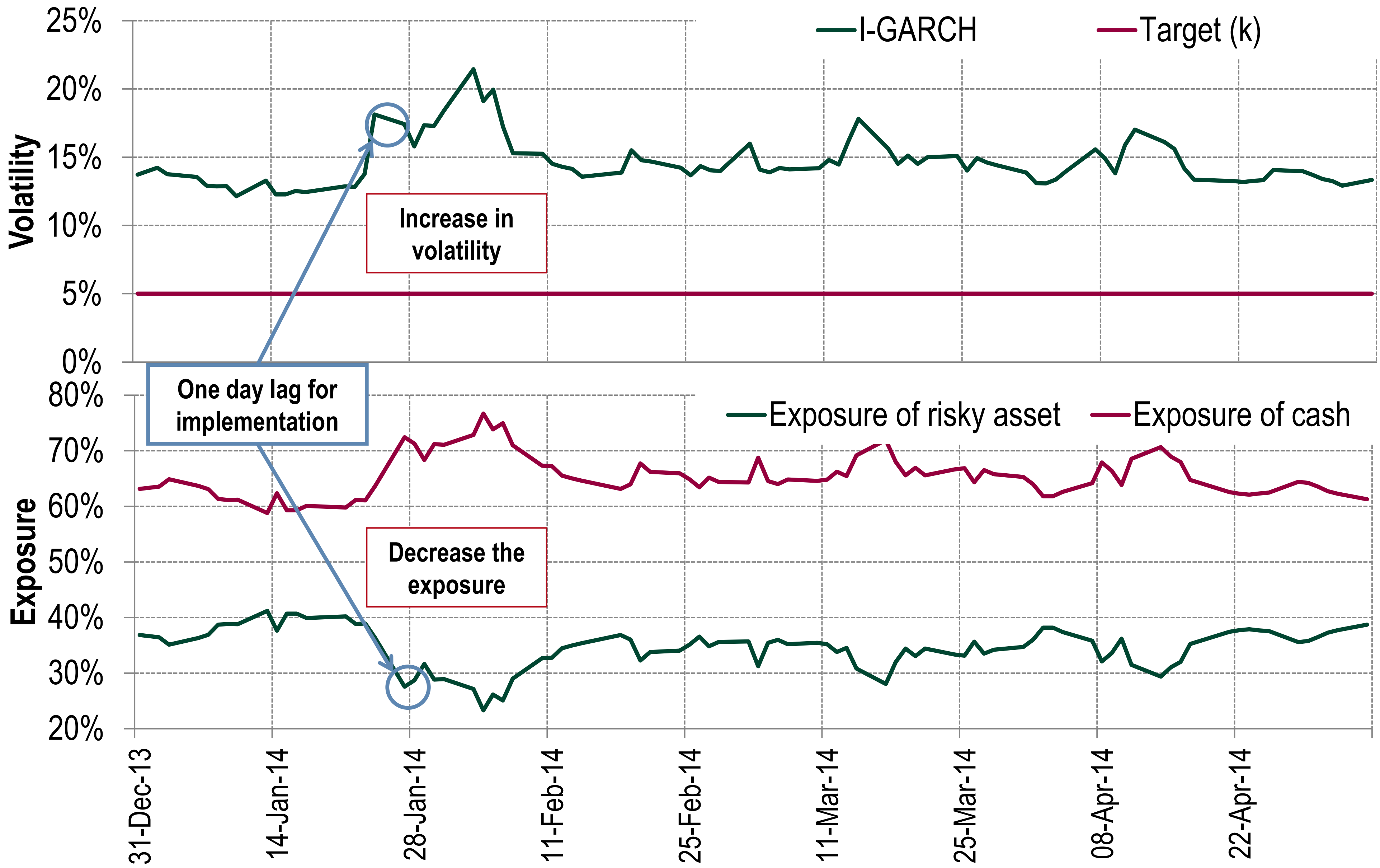
Inter temporal risk parity strategy in practice



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If returns of risky assets had Gaussian distributions

Gaussian distributed returns	Buy and Hold	Inter-temporal Risk Parity
Average annualized excess return	7.5%	7.7%
Average annualized volatility	18.8%	19.3%
Sharpe ratio	0.40	0.40
Maximum drawdown (MDD)	-37.6%	-38.5%
Ratio MDD / volatility	-2.0	-2.0
Average exposure	100.0%	101.8%*
Improvement in Sharpe ratio	-	0.00
Std Dev of improvement in Sharpe ratio	-	3.5%

Source: R Perchet, R Leote de Carvalho, T Heckel and P Moulin, "Inter-temporal risk parity: A constant volatility framework for equities and other asset classes" (2014) http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2384583

Substantial effort for nothing. But, before transaction costs, no loss either..

- Returns of risky assets do not follow Gaussian distributions: clustering, fat tails, leverage effects, …
  - Rama Cont, "*Empirical Properties of Asset Returns: Stylized Facts and Statistical Issues.*" Quantitative Finance, 1 (2001), 223-236

* Average of 1 / σ_t > 1 for an uniform function, thus average exposure > 100%

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Inter-temporal risk parity applied to equities

- Evidence that managing equities at constant risk adds value:
  - Hocquard, Ng and Papageorgiou (2013)
  - Cooper (2010)
  - Kirby and Ostdiek (2012)
  - Ilmanen & Kizer (2012)
  - Giese (2012)

Higher Sharpe ratio and smaller drawdowns with constant volatility portfolio..

- But no consensus regarding where added value comes from:
  - Hallerbach (2012)

Better volatility forecast and less variability in volatility is sufficient to improve Sharpe ratio.

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Understanding inter-temporal risk parity strategies

Monte Carlo simulations with scenarios generated from parametric models

- Apply different stochastic models [1] for risky asset returns
  - Keep risk premium μ constant over time
- Apply different volatility models [2]
  - GARCH family of models
  - Introduce effects, i.e. leverage effect
- Different noise [3]
  - Gaussian
  - t-student for higher probability of fat tail events
  - skewed for larger extreme events

1 $$r_t = \mu + \sigma_t z$$

2 $$\sigma_t{}^2 = \omega + \alpha(r_t - \mu)^2 + \beta\sigma_{t-1}{}^2$$

ω long-term volatility level
α volatility clustering
higher alpha => larger clustering effect
β persistency of past volatility
~ 1 => few changes in the day-to-day volatility
$\alpha+\beta$ must be < 1 for stationarity
Features like leverage effect i.e. volatility more impacted by negative returns can also be added

3 $$Z \sim N(\ldots)$$

Compare buy and hold with the average behaviour observed over many simulated scenarios.

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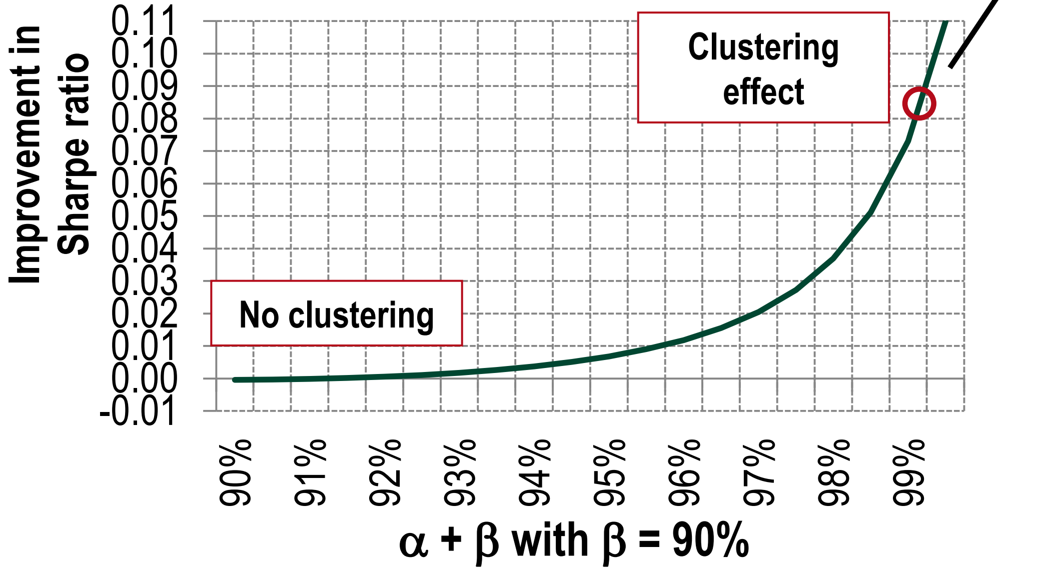
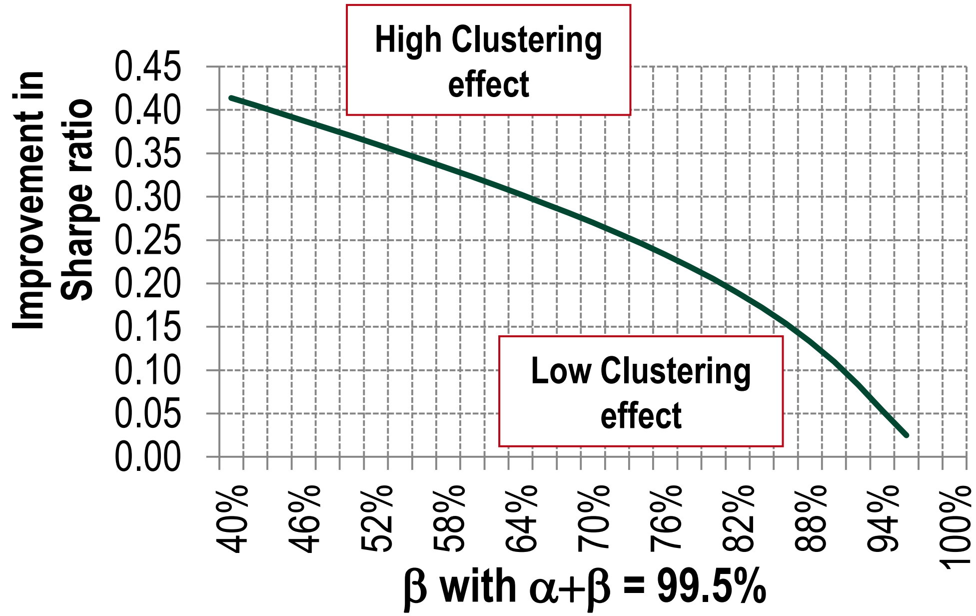
Volatility clustering explains better risk-adjusted performances

Using standard GARCH model

- Generate volatility clustering while keeping risk premium is constant:
  - Higher Sharpe ratio in lower volatility regimes
  - Lower Sharpe ratio in higher volatility regimes
- Clustering of volatility adds predictability while:
  - Increased exposure in lower volatility regimes
  - Decreased exposure in higher volatility regimes
- Market timing effect!

GARCH with α = 9% and β = 90%	Buy and Hold	Inter-temporal Risk Parity
Average annualized excess return	7.5%	9.1%
Average annualized volatility	18.8%	18.9%
Sharpe ratio	0.40	0.48
Average exposure	100.0%	121.6%
Improvement in Sharpe ratio	-	0.08
Std Dev of improvement in Sharpe ratio	-	11.4%





Source: R Perchet, R Leote de Carvalho, T Heckel and P Moulin, "Inter-temporal risk parity: A constant volatility framework for equities and other asset classes" (2014)

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Fat tails, leverage effect and skew

- Fat tails (GARCH with t-student noise)
  - – Increase the probability of extremes events
  - ⇒ Improvement of the Sharpe ratio
  - ⇒ Reduces largest drawdown events
- Leverage effect (GJR-GARCH):
  - – Volatility increases more with negative returns, i.e. negative correlation between volatility and returns
  - ⇒ Reduces largest drawdown events
- Larger negative return (Skewed-GARCH)
  - – Increase probability of larger negative return
  - ⇒ Reduces largest drawdown events

Inter-temporal risk parity strategy improves returns and filters out fat tails thanks to predictability of volatility due to clustering. Negative correlation between return volatility add to the benefit.

GARCH with t-Student noise	Buy and Hold	Inter-temporal Risk Parity
Average annualized excess return	7.4%	10.3%
Average annualized volatility	17.8%	18.8%
Sharpe ratio	0.41	0.55
Maximum drawdown (MDD)	-37.2%	-35.2%
Ratio MDD / volatility	-2.1	-1.9
Improvement in Sharpe ratio	-	0.13
Std Dev of improvement in Sharpe ratio	-	13.9%

	GJR-GARCH		Skewed-GARCH	
	Buy and Hold	Inter temporal Risk Parity	Buy and Hold	Inter temporal Risk Parity
Average annualized excess return	7.7%	9.4%	7.2%	9.0%
Average annualized volatility	19.1%	18.8%	18.4%	18.9%
Sharpe ratio	0.40	0.50	0.39	0.48
Maximum drawdown (MDD)	-42.7%	-38.3%	-38.7%	-36.8%
Ratio MDD / volatility	-2.2	-2.0	-2.1	-2.0
Improvement in Sharpe ratio	-	0.10	-	0.08
Std Dev of improvement in Sharpe ratio	-	13.4%	-	11.9%

Source: R Perchet, R Leote de Carvalho, T Heckel and P Moulin, "Inter-temporal risk parity: A constant volatility framework for equities and other asset classes" (2014)

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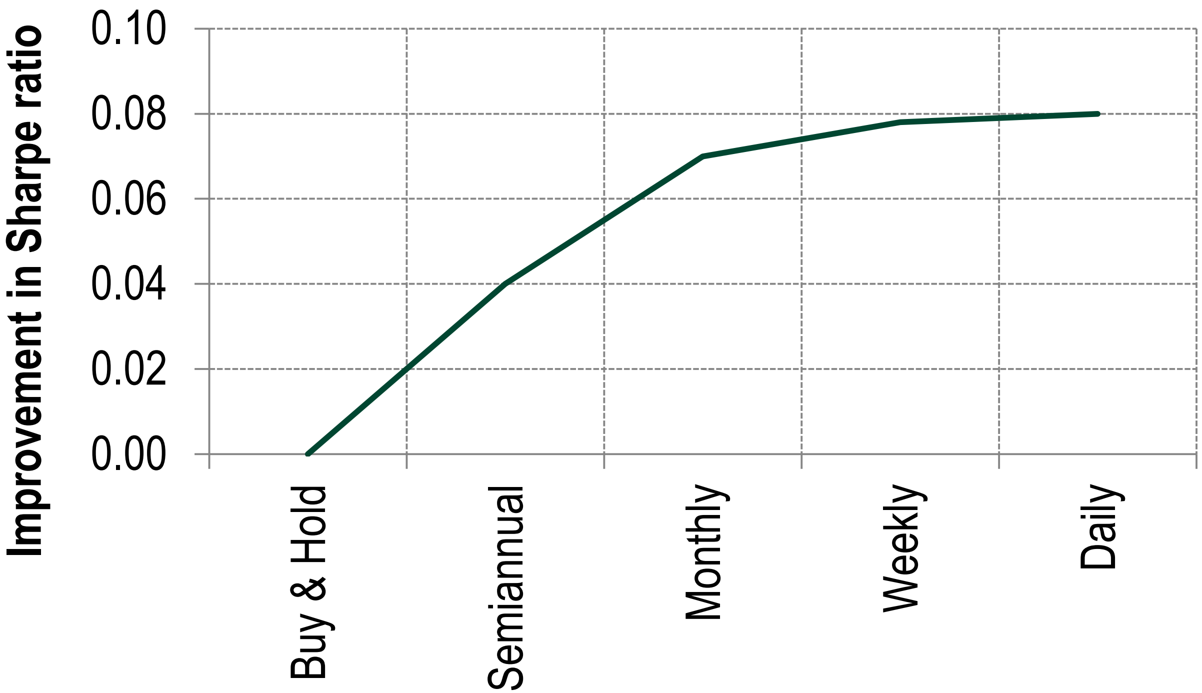
Impact of rebalancing frequency



Source: R Perchet, R Leote de Carvalho, T Heckel and P Moulin, "Inter-temporal risk parity: A constant volatility framework for equities and other asset classes" (2014) http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2384583

- At weekly re-balancing, the benefits from an inter-temporal risk parity strategy remain strong

Lower frequency means substantially lower turnover.
Optimal strategy with daily monitoring and rebalancing only when significant changes are observed.

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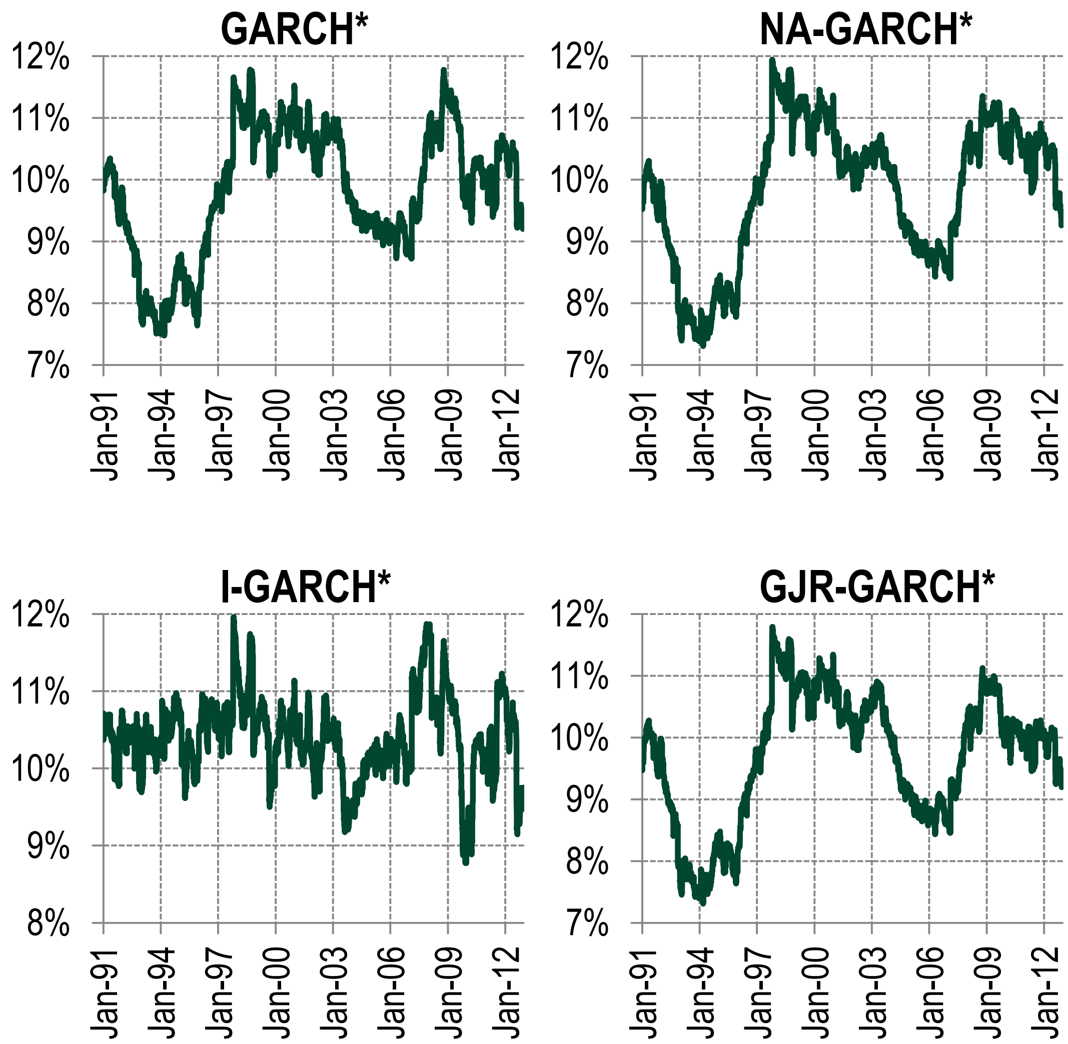
Forecasting volatility (S&P500)

- Different GARCH models considered
  - Target 10% volatility ex-ante
- GARCH which includes
  - Volatility clustering
  - Long term volatility
- NA-GARCH & GJR-GARCH which include
  - Volatility clustering
  - Long term volatility
  - Leverage effect
- I-GARCH which includes
  - Volatility clustering
- 1-year ex-post volatility is measured

I-GARCH model does best at forecasting volatility



* Comparison of the 1-year rolling ex-post volatility for the inter-temporal risk parity strategy applied to the S&P 500. The target volatility is 10% and the forecast volatility is based on four different GARCH models with parameters estimated from an expanding window once every year at the start of each year. *Source: R Perchet, R Leote de Carvalho, T Heckel and P Moulin, "Inter-temporal risk parity: A constant volatility framework for equities and other asset classes" (2014)*

Larger clustering effect in riskier asset classes

- Estimation of clustering effect and fat tail events for main asset classes
  - Large α for Equities, in particular Emerging, and for US high yield
    more volatility clustering
  - Smaller α for government bonds and for investment grade bonds
    less volatility clustering
- $\alpha + \beta \sim 1$ for all assets
  - most of the volatility explain past volatility and new events
- Very small impact of long term volatility
- High probability of extremes events in US high yield and Russell 1000

	Russell 1000	MSCI Emerging Markets	S&P GSCI Commo.	US high yield Bonds	US Invest. Grade bonds	US 10Y Gov. Bonds
ω	7.0E-7	1.6E-6	8.0E-7	2.0E-7	1.0E-7	3.0E-7
α	6.1%	9.6%	5.4%	21.7%	4.0%	4.3%
(t-stats)	(9.90)	(11.4)	(11.0)	(12.7)	(8.7)	(8.7)
β	93.3%	89.3%	94.1%	75.7%	95.0%	94.5%
(t-stats)	(132.3)	(89.5)	(171.9)	(38.0)	(162.1)	(136.7)
$\alpha + \beta$	99.4%	98.9%	99.5%	97.4%	99.0%	98.9%
t-Student	5.6	7.1	7.4	3.7	6.7	7.6
(t-stats)	(16.8)	(13.2)	(12.7)	(33.9)	(12.8)	(12.7)

Source: R Perchet, R Leote de Carvalho, T Heckel and P Moulin, "Inter-temporal risk parity: A constant volatility framework for equities and other asset classes" (2014) http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2384583

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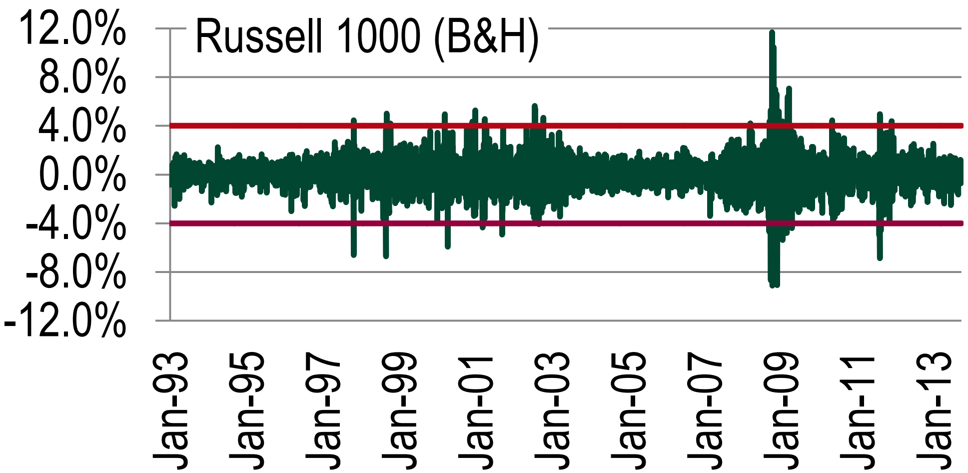
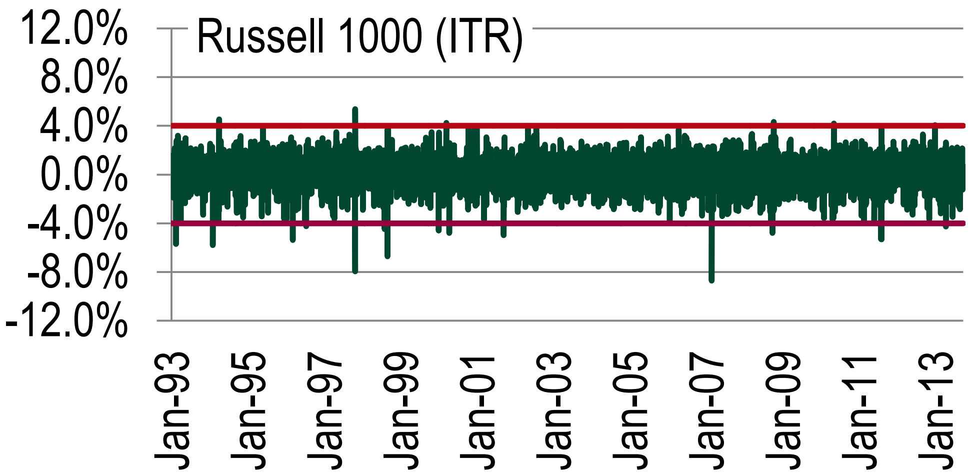
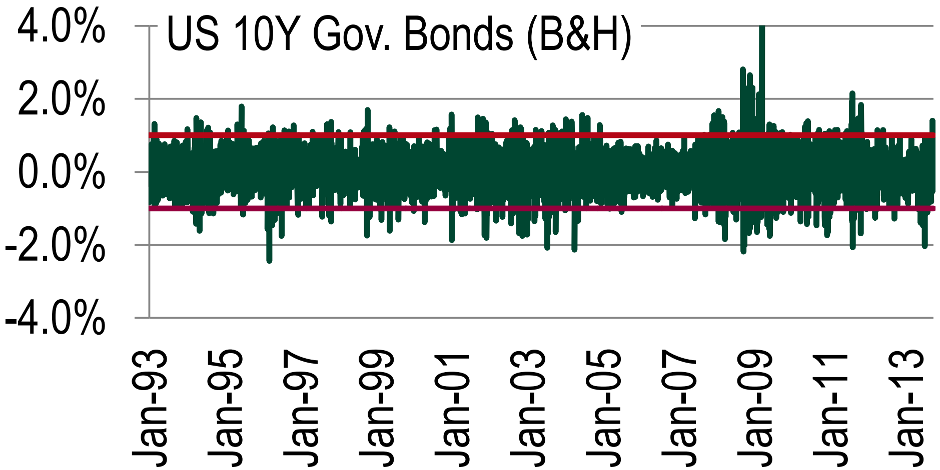
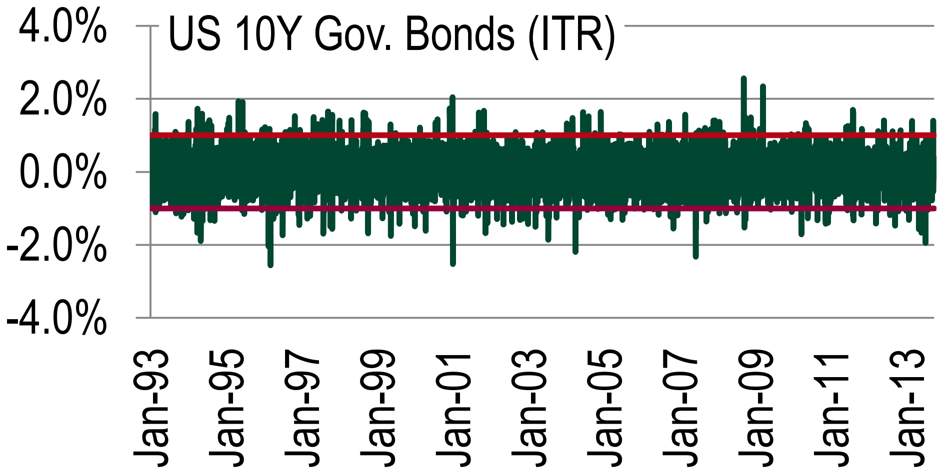
Chunnel returns with Inter-temporal risk parity strategy









Historical simulations for different asset classes

- Inter-temporal risk parity strategy applied to equity indices and other asset classes
  - Higher Sharpe ratio for asset classes with stronger volatility clustering and fat tails
    - High yield bonds
    - Emerging Equities
    - Developed Equities
    - Less for commodities
  - Corporate bonds and government bonds
    - Low clustering in the last 20 years
    - No significant benefit

* Comparison of a buy-and-hold strategy for different asset classes with inter-temporal risk parity strategies based on historical simulations. The target volatility was set at 5%. Volatility forecasts as based on I-GARCH models. The I-GARCH model parameters were estimated from an expanding window once every year at the start of each year.

Source: R Perchet, R Leote de Carvalho, T Heckel and P Moulin, "Inter-temporal risk parity: A constant volatility framework for equities and other asset classes" (2014) http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2384583

	Russell 1000	MSCI EM Markets	S&P GSCI Commo	US high yield Bonds	US Invest. Grade bonds	US 10Y Gov. Bonds
	Buy and hold strategy					
Average annual. Excess return	8.0%	6.7%	2.3%	4.8%	3.7%	3.2%
Average annual. Volatility	19.0%	19.2%	21.6%	4.4%	5.1%	8.0%
Sharpe ratio	0.42	0.35	0.11	1.09	0.73	0.40
Maximum drawdown (MDD)	-55.8%	-65.2%	-73.4%	-29.1%	-16.7%	-14.1%
Ratio MDD / volatility	-2.9%	-3.4	-3.4	-6.6	-3.3	-1.8
I-GARCH	Inter-temporal risk parity strategy					
Average annual. Excess return	2.9%	3.0%	0.8%	8.5%	3.9%	2.1%
Average annual. Volatility	5.2%	5.4%	5.2%	5.5%	5.1%	5.2%
Sharpe ratio	0.56	0.56	0.15	1.55	0.76	0.40
Maximum drawdown (MDD)	-10.4%	-19.1%	-16.7%	-28.5%	-11.2%	-10.2%
Ratio MDD / volatility	-2.0	-3.5	-3.2	-5.2	-2.2	-2.0
Improv. in Sharpe ratio	0.14	0.21	0.05	0.45	0.04	0.00

Factor investing

- Factor investing has been gaining attention since Fama & French (1992,1993)
  - Value and Size premiums in equity markets
- Carhart (1997) extended Fama and French model
  - Momentum premium was added
- Qian, Sorensen and Hua (2009) found value premium in other asset classes
  - Government bonds
  - Foreign exchange
- Asness, Moskowitz and Pedersen (2013) generalize value and momentum premiums
  - Government bonds
  - Foreign exchange
  - Commodities
- Capture premiums: long-short portfolios
  - E.g. long the cheapest securities and short the most expensive securities

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Value and Momentum premiums

Inter-temporal risk parity strategy applied to Value and Momentum factors:

- Equities: daily data from Ken French's web-site:
  - Value premium: HML (High-Minus-Low factor)
  - Momentum premium: Mom (Momentum)
- Sovereign Government bonds based on 10 countries*:
  - Value premium: slope of the yield curve (10-year bond yields minus cash rates)
  - Momentum premium: past twelve month cumulative returns of total return indices
- Foreign exchange based on 10 countries**:
  - Value premium: carry strategy using inter-bank rates
  - Momentum premium: past twelve month cumulative returns of forward returns

* Australia, Canada, Germany, Japan, Denmark, Norway, Sweden, Switzerland, UK and US
** Australia, Canada, Germany or Euro zone after 1999, Japan, New Zealand, Norway, Sweden, Switzerland, UK and US

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Improvement of information ratios in factor investing

Applying inter-temporal risk parity to factor investing also bring benefits

- Improvement of information ratios
- Larger impact for underlying risky asset classes
  - Equities and foreign exchange
- Lower impact for government bonds
- Also robust to rebalancing frequency
  - Weekly or monthly rebalancing

* Comparison of a buy-and-hold strategy for different factor with inter-temporal risk parity strategies based on historical simulations. The target volatility was set at 5%. Volatility forecasts as based on I-GARCH models. The I-GARCH model parameters were estimated from an expanding window once every year at the start of each year. Bloomberg, BNP Paribas Investment Partners, January 2014

Source: R Perchet, R Leote de Carvalho and P Moulin, "Inter-temporal risk parity: An application to factor investing." (2014)

	Momentum Equity	Value Equity	Momentum Foreign Exchange	Value Foreign Exchange	Momentum Fixed Income	Value Fixed Income
	Buy and hold strategy					
Average annual. Excess return	8.4%	3.3%	1.7%	3.9%	-0.3%	3.1%
Average annual. Volatility	14.1%	9.5%	9.1%	8.8%	5.7%	6.1%
Information ratio	0.59	0.34	0.19	0.44	-0.06	0.52
Maximum drawdown (MDD)	-63.0%	-44.5%	-27.8%	-34.0%	-28.9%	-11.4%
Ratio MDD / volatility	-4.5	-4.7	-3.0	-3.9	-5.1	-1.9
I-GARCH	Inter-temporal risk parity strategy					
Average annual. Excess return	7.7%	2.2%	2.5%	3.3%	0.8%	2.9%
Average annual. Volatility	5.4%	5.3%	5.3%	5.3%	5.2%	5.2%
Information ratio	1.43	0.42	0.46	0.63	0.16	0.57
Maximum drawdown (MDD)	-13.9%	-22.1%	-14.9%	-17.1%	-18.1%	-9.1%
Ratio MDD / volatility	-2.6	-4.2	-2.8	-3.2	-3.5	-1.7
Improv. in information ratio	0.83	0.08	0.27	0.19	0.22	0.05

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Conclusions

- No Gaussian behavior or returns explains why constant volatility strategy add value
- Investors should think in terms of risk budget allocation rather than fixed weights
- Improvement of Sharpe ratio and information ratio explained by volatility clustering
  - Volatility is not constant over time and is predictable to some extent
- Presence of fat tails events increase volatility clustering effect
- Benefit of risk management is larger if return and volatility are negatively correlated
- Clear benefit for risky asset classes: equities, high yield and foreign exchange rates
- Less added value but keep the risk exposure under control for less risky asset classes such as government bonds

Our research show how risk management can improve risk-adjusted returns!

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