Random Errors and Systematic Returns
or
The Flaw in Fundamental Prices

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Overview

Key Questions
1. Can random errors explain above average returns of certain portfolios?
2. Why? What does this say about the nature of equilibrium prices?
3. How does it relate to risk based explanations?
4. What are the practical implications?

Along the way, touch on:
- The Stein Paradox
- Fama French ‘93 & Berk’s ’95 critique
- The fundamental indexing discussion (are cap weighted portfolios overpriced?)
Random Errors and the Fundamental Indexing Debate

<table>
<thead>
<tr>
<th>Fundamental Indexer Claims</th>
<th>Opponent's Claims</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Pricing errors =&gt; Cap weighted PF is overvalued</td>
<td>• Cap weighted portfolio is correctly priced</td>
</tr>
<tr>
<td>• Occasional corollary: errors explain size and value effects</td>
<td>• No secondary effects unless signal information is not correctly priced in</td>
</tr>
<tr>
<td>• This is necessarily the case</td>
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This Presentation:

• Everybody has a (partial) point
• This can be resolved by looking at prices, not returns
• The resolution has implications for many “risk premia” and “smart beta” strategies (more than even the fundamental indexers claim)
What Is a Pricing Error?

1. A company = Random Cash Flow Generator

2. DEFINITIONS:
   - Parameters Known => Intrinsic Value
   - Parameters Estimated => Market Value
   - Difference: Pricing Error

3. INITIAL ASSUMPTION: Errors are Pure Noise.
   - I.e. Market Caps are unbiased company-by-company “fundamental estimates”

   \[ M = I \times Z \]
   \[ m = m + z \]

   all variables \sim \text{lognormal}
   all variables \sim \text{normal}

Note: Presence of Error by itself not at odds with market efficiency
First, Noise-Like Errors Do not Cause the Cap Weighted Portfolio to Be Mispriced

Example: Two companies both with intrinsic value of $10 and Pricing Error

<table>
<thead>
<tr>
<th>STOCK 1</th>
<th>STOCK 2</th>
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<tbody>
<tr>
<td>Intrinsic: $10</td>
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</tr>
<tr>
<td>Market: $5</td>
<td>$15</td>
</tr>
</tbody>
</table>

\[ \text{Cap is 50\% low} \]
\[ \text{Cap is 50\% high} \]

I. CAP Weighted PF

Position in $10 PF:

\[ \text{\$2.50} \]

Intrinsic value of PF:

\[ \frac{\$2.50}{\$5} \times \$10 = \$5 \]

\[ \frac{\$7.50}{\$15} \times \$10 = \$5 \]

\[ \text{\$10} \]

In fact, the cap weighted PF is perfectly insulated against relative mispricing
However, Could Noise Cause the 1/N and Size Effects?

Again: Two companies both with intrinsic value of $10 and Pricing Error

**II. EQUAL WEIGHTED PF**

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Cap is 50% low

Cap is 50% high

Investment in $10 PF: $5

Intrinsic value of PF:

$5 / $5) * $10 = $10

$5 / $15) * $10 = $3.33

= $13.33

The equal weighted portfolio appears under-valued. Do random errors cause a size effect?

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What’s Going On?

Random errors are now easily exploitable

Note: This will inevitably happen if prices really are “intrinsic values + uncorrelated error”

Translated into the fundamental indexing debate:

• Fundamental Indexers: “That’s just how it is”
• Opponents: “This can’t be what prices are like”

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How to Fix It?

**INTRINSIC VALUES (ln logs)**

- Small cap, Correctly priced
- Large cap, Correctly priced

**MARKET VALUES = INTRINSIC VALUES + UNCORRELATED ERROR**

Aka “Fundamental Estimates”

- Small Caps will tend to be too cheap
- Large Caps will tend to be too expensive

**BAYES!**

\[ m' = \frac{\text{Var}(v)}{\text{Var}(m)} m \]

**SYSTEMATICALLY EXPLOITABLE: SIZE EFFECT**

**UN-EXPLOITABLE NO SIZE EFFECT**

**WHAT TO DO TO REMOVE SYSTEMATIC ERROR?**

SHRINK ALL VALUES TOWARDS THE COMMON MEAN.

MAYBE BY BETTING ON SIZE FACTOR?

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The Resolution: In Efficient Markets, Prices Need to Be “Bayesian”

**UNADJUSTED MARKET VALUES = INTRINSIC VALUES + ERROR**

**EXPLOITABLE**

Raise the price of small caps

Lower the price of large caps

**BAYSIAN ADJUSTED MARKET VALUES**

**UN-EXPLOITABLE**

KEY PROPERTIES OF CORRECTLY ADJUSTED PRICES

1) Prices still have errors
2) However, errors are now uncorrelated with market values, not intrinsic values
3) Prices are no longer the unbiased “fundamental” estimate of intrinsic value!
4) As a result, errors are un-exploitable
   ⇒ This is in essence the scenario described in Perold 2008
   ⇒ The logic is the same as in Stein’s Paradox (1956*)
The Resolution: In Efficient Markets, Prices Need to Be “Bayesian”

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*“Inadmissibility of the Usual Estimator for the Mean of a Multivariate Normal Distribution”
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Is this what actually happens?

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Are Prices Correctly Adjusted? It’s Not Clear

Adjustments are **difficult**…

- Need to know that there are pricing errors
- How are they measured?
  - No direct way
  - Indirectly? maybe via historic size effect?
- Does the magnitude change over time? Has it already been adjusted for?

…and possibly **risky**

- See J. Stein on the events of 8/2007: Fundamental investors vs quants
- Quants fix mispricing via (risky) factor bets

So, **are prices correct empirically?**

- Derive implications
- Look at evidence
Implications Encompass Many “Risk Premia”

Class I: Market Cap (see above)

Class II: Cap-by-X:
- Value ratios (M/B, Price/dividend, Price/earnings, CAPE)
- Other X: Market cap per employee, market cap by expenses
- Price per share!

These should be stronger than pure market cap sorts, provided X absorbs some of the cap variation without being subject to the same valuation error.

Class III: Error Magnitudes - Anything that separates high and low error stocks
- “Quality” (earnings volatility, cash flow volatility, sales volatility)
- “Vol” idiosyncratic return volatility

=> This goes far beyond just Size and Price/Book
Empirical Results: Lots of Evidence in Favor of the Thesis

“Sorts”

- Size (and 1/n): Banz 1981
- Value: Rosenberg, Reid and Lanstein 1985
- Price-per-Share subsumes most of size: Kross 1985
- Quality & Analyst Dispersion (Zhang 2006)
- Ivol (Ang et al 2006)
- These are early papers on each effect. There are many others.

Related symptoms in long-only portfolios

- 1/n
- Fundamentally weighted portfolios
- Low volatility and low beta portfolios
- Quality portfolios
- Several “Smart Beta” flavors
Two Particularly Telling Pieces of Evidence

1. The Size Effect Is Mostly a Price Effect
e.g. Kross (1985)

\[
\text{Market Cap} = \text{Log(Shares Outstanding)} + \text{Log(Price/Share)}
\]

- 25% of explanatory power
- 75% of explanatory power

- This is not easily explained with a pure risk story or otherwise

2. Size vs Ivol Sort “The Sign Reversal”
Bollen et al. (2008)

- This sign reversal is very specific to the pricing error story
- Similar results can be found in Bali and Cakici (2008) and Chen et al (2012)
Before Drawing Conclusions, Consider a Completely Different Angle

Fama French & Berk’s Critique; “Risk Discounts”

- **Fama French ‘93**: size and value stand out empirically. Therefore they are risk factors.
- **Berk ‘95**: of course they stand out. They have to:
  - Companies are not risky because they are small. They have small market caps (and small P/B) because they are risky.
  - Evidence: Fundamentally small companies are actually not riskier than others.

=> This is a key insight! It also turns the traditional story upside down.

**Prices without any price discounts for risk (same expected return for all)**

**Prices with price discounts and price premia**

*Blue stocks are exposed to risks, red stocks are not*
Comparison: Poorly Adjusted Random Errors, or Risk?

<table>
<thead>
<tr>
<th></th>
<th>Pricing Errors</th>
<th>Risk Discounts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Value</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Comovement</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>EMH compatibility</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>Size not subsumed by Value</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Joint Sharpe is too high</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>Price per Share (subsumes Size)</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Ivol (incl sign reversal)</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>Quality, Analyst Dispersion</td>
<td></td>
<td>✓</td>
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=> Very likely it’s both

*EMH = efficient market hypothesis
Implications for Investors

I. A framework to judge what’s a risk factor and what’s a mistake

1) Size, Value, Value-variants (“Price by X’’): More likely risk factors plus a mistake in the historic sample
2) Low Volatility and Quality: More likely inefficiencies

II. And therefore what to reasonably expect / what to ask yourself

1) Likely weakening relative to deep history. Not uniform.
2) Risk factors: How good are your nerves?
   • Some contain macro risk (see great recession drawdowns)
3) Other factors: Do you want to be in the “arbing game”?
   • Crowding and mispricing risk (see August 2007)
   • Look for a warning system for “over-arbing” (Note: this wouldn’t matter under EMH)
Example: Dividends – A Value Effect plus a Mistake?

High Dividend Yield used to be cheap

By 2013 its relative price had risen. It also became popular

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Factor Valuations; Another Example

Vol in the S&P 500

Price/Book Ratios of Quintiles formed on Volatility

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Source: Geode Capital Management. For illustrative purposes only.
The universe is the S&P 500 Index.

High vol isn’t usually that relatively cheap outside of recessions
Factor Valuations; Another Example

Vol in EAFE

Price/Book Ratios of Quintiles formed on Volatility

Ratio of Price/Book Ratios: Top vs. Bottom Volatility Quintile

In EAFE high vol is fairly rich

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Summary

The Theory:

• Random errors are a candidate explanation for many common strategies: Size, Value, Quality, Low Vol etc.

• Empirical evidence suggests that they coexist with risk as a key driver

Some Implications:

• Factor behavior is expected to differ on conceptual grounds (not just empirically). This should be taken into account when designing alts portfolios and setting expectations

• Regardless of the exact theory - if it’s not strictly about risk, factor crowding and pricing become very important

• Key ideas can be extended to other asset classes

Low error stocks (low vol, high quality)

High error stocks (high vol, low quality)

1) Among non-micro cap, High Error Stocks will be more overvalued.

2) Oddly, It’s the opposite for micro caps

=> Low volatility and quality will outperform

Note: traditional explanation is “limits to arbitrage”
What’s going on? In different words

\[ A + B = C \]

- A and B are uncorrelated, mean-zero random variables.
- Only their sum C can be observed. A and B can’t

If we observe a large positive value for C, what is our best guess for B? It’s:

\[ E(B \mid C > 0) > 0 \] !!

- In the random error model, C is the market cap, A is the intrinsic value and B is the error (all in logs with market cap demeaned)

If we observe above average market cap and the error is uncorrelated with intrinsic value, then our best guess for the error is positive, not zero

Therefore the price has to be adjusted so that the error is something other than uncorrelated noise around intrinsic value. The unbiased fundamental estimate is not the correct price.

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Note 2: Gross performance is that of each relevant fund/composite managed by Geode during the period in question. Performance shown represents the asset-weighted gross returns with new accounts added during its first full month. Gross performance results are net of trading commissions. The performance results are gross of investment management or incentive fees and therefore the results do not reflect the deduction of such fees. The performance results do bear other fees (such as administration fees), which have not been deducted from these results. When compounded over a period of years, fees would decrease returns. This hypothetical example illustrates the compound effect of fees on investment return: If a portfolio’s annual rate of return is 15% for 5 years and the annual management fee is 100 basis points, the gross cumulative five year return would be 101.1% and the five-year return net of fees would be 92.5%. Performance results are expressed in US dollars. This presentation of unaudited gross performance is only intended for one-on-one presentations with clients, prospects and consultants with whom Geode has a pre-existing relationship and may not be duplicated in any form by any means or re-distributed without our prior written consent. Information about our investment advisory fees are described and available in FORM ADV Part 2A.

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