A cross-section across CVA

19 FEBRUARY 2013

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Abstract

For calculating advanced CVA VaR capital requirement, Basel III states that, where the counterparty does not have liquid CDS spreads, financial institutions shall use a proxy spread level that is appropriate having regard to the rating, region and industry of the counterparty. Additionally the European Banking Authority (EBA) has been laying down further rules as part of CRDIV about the way in which proxy spreads should be calculated. In this paper, we review the intersection model proposed by the EBA. We propose an alternative approach based on a cross-sectional regression that has the advantage of providing more robust and stable results without loss of transparency. In contrast to the intersection model, our proposed cross-section model generates realistic historic spread levels which could be used for CVA risk capital and P&L.

1. Introduction

While Basel II covers the default risk for capital requirement, the Bank for International Settlements (BIS) recognizes that mark-to-market losses associated with a deterioration of credit worthiness of counterparty are not captured. Given that during the financial crisis it was an important source of losses, BIS proposes in addition a capital requirement to cover losses on Credit Valuation Adjustment (CVA) to over-the-counter derivatives trading.

This additional capital requirement is defined as the sum of the Value at Risk (VaR) and the stressed Value at Risk on a 99% percentile and a 10-day time horizon, times a three multiplier. In addition this calculation is done by segregating CVA VaR from trading VaR and restricting VaR to changes to counterparties' credit spreads only. BIS proposes two methods: A standardized method and an advanced method. The standardized method is applied for institutions with neither Internal Model Method (IMM) nor VaR model approvals. It does not require any modelling and is based on current counterparty exposure, maturity, and external rating. The advanced method requires having an exposure approved model. It requires defining the CVA risk to the counterparty credit spread and computing VaR on historical credit spread time series.

For the advanced method, when the credit spread is not liquid and, hence, not observable in the market, institutions are required to proxy the credit spread having regard to rating, region, and sector of the counterparty. The EBA proposed methodology[1] is to average data of liquid names across the relevant rating, region, and sector sub-categories to imply the proxy spread of illiquid names. This is defined as the Intersection or Bucket method.

Our proposal is to use instead a cross-section[2] methodology that is based on a multi-dimensional regression across rating, region and industry sector. This method is avoids many of the stability, robustness, and consistency problems associated with the intersection methodology prescribed in the EBA proposal.

This paper describes the cross-section methodology, and explains why we believe it to be preferable to an approach based on intersections of categories. In section 2, we describe the intersection methodology proposed by the EBA. In section 3, we define the cross-section model in the context of credit spreads and discuss the calibration of the model based and present the results based on liquid spread from Markit. In section 5, we show the stability, robustness, and consistency of the cross-section model compared to the intersection model.

[1] *EBA, risk (July 2012) "Technical standards in relation with credit valuation adjustment".* *http://www.eba.europa.eu/Publications/Consultation-Papers/All-consultations/2012/EBA-CP-2012-09.aspx*

[2] *Rosenberg Barr, Michael Houglet and Vinay Marathe (1974) "Extra Market Components of Covariance In Security Returns" JFQA.*

2. The intersection methodology

In the intersection methodology, the proxy spread for a given obligors shall be determined by aggregating data across the relevant rating, region, and sector sub-categories forming a bucket. Then the proxy spread of obligor i is defined as:

$$S_i^{proxy} = \frac{1}{N}\sum_{j=1}^{N} S(j)$$

where $N \geq 1$ is the number of liquid names in the same rating, region, sector sub-categories as obligor i and $S(j)$ is their spread levels.

Markit provide a daily file of liquid CDS spreads and recovery rates, together with the number of contributors. The file also contains sectors, regions and ratings. This gives us a high-quality and independent data source for calibrating the proxy spread factors. Based on Markit data and the EBA's minimum granularity requirement, Table [1] provides details of the different sub-categories on 18 February 2013. The first three columns provide rating, region, sector and the last column contains the number of liquid names in this specific bucket.

Table [1]: Number of liquid obligors by sector/region/rating, using Markit data from 18 February 2013

Sector	Region	Rating	Num Obligors
Financial Services	North America	AA-AAA	8
Financial Services	North America	A	40
Financial Services	North America	BBB	58
Financial Services	North America	BB	15
Financial Services	North America	B	7
Financial Services	North America	CCC	5
Financial Services	Europe	AA-AAA	12
Financial Services	Europe	A	62
Financial Services	Europe	BBB	34
Financial Services	Europe	BB	17
Financial Services	Europe	B	7
Financial Services	Europe	CCC	4
Financial Services	Japan	AA-AAA	1
Financial Services	Japan	A	10
Financial Services	Japan	BBB	10
Financial Services	Japan	BB	0
Financial Services	Japan	B	1
Financial Services	Japan	CCC	0
Financial Services	Asia ex-Japan	AA-AAA	8
Financial Services	Asia ex-Japan	A	25
Financial Services	Asia ex-Japan	BBB	10
Financial Services	Asia ex-Japan	BB	0
Financial Services	Asia ex-Japan	B	0
Financial Services	Asia ex-Japan	CCC	0
Financial Services	Rest of World	AA-AAA	0
Financial Services	Rest of World	A	8
Financial Services	Rest of World	BBB	6
Financial Services	Rest of World	BB	0
Financial Services	Rest of World	B	0
Financial Services	Rest of World	CCC	0
Non-Financial Services	North America	AA-AAA	1
Non-Financial Services	North America	A	20
Non-Financial Services	North America	BBB	44
Non-Financial Services	North America	BB	24
Non-Financial Services	North America	B	27
Non-Financial Services	North America	CCC	9
Industrial Production	Asia ex-Japan	AA-AAA	1
Industrial Production	Asia ex-Japan	A	2
Industrial Production	Asia ex-Japan	BBB	6
Industrial Production	Asia ex-Japan	BB	4
Industrial Production	Asia ex-Japan	B	0
Industrial Production	Asia ex-Japan	CCC	0
Industrial Production	Rest of World	AA-AAA	1
Industrial Production	Rest of World	A	0
Industrial Production	Rest of World	BBB	0
Industrial Production	Rest of World	BB	0
Industrial Production	Rest of World	B	0
Industrial Production	Rest of World	CCC	0
Raw Materials	North America	AA-AAA	0
Raw Materials	North America	A	7
Raw Materials	North America	BBB	27
Raw Materials	North America	BB	11
Raw Materials	North America	B	1
Raw Materials	North America	CCC	0
Raw Materials	Europe	AA-AAA	0
Raw Materials	Europe	A	7
Raw Materials	Europe	BBB	13
Raw Materials	Europe	BB	8
Raw Materials	Europe	B	3
Raw Materials	Europe	CCC	1
Raw Materials	Japan	AA-AAA	0
Raw Materials	Japan	A	6
Raw Materials	Japan	BBB	11
Raw Materials	Japan	BB	4
Raw Materials	Japan	B	0
Raw Materials	Japan	CCC	0
Raw Materials	Asia ex-Japan	AA-AAA	0
Raw Materials	Asia ex-Japan	A	3
Raw Materials	Asia ex-Japan	BBB	3
Raw Materials	Asia ex-Japan	BB	0
Raw Materials	Asia ex-Japan	B	0
Raw Materials	Asia ex-Japan	CCC	0

Sector	Region	Rating	Num Obligors
Non-Financial Services	Europe	AA-AAA	1
Non-Financial Services	Europe	A	10
Non-Financial Services	Europe	BBB	43
Non-Financial Services	Europe	BB	15
Non-Financial Services	Europe	B	10
Non-Financial Services	Europe	CCC	0
Non-Financial Services	Japan	AA-AAA	7
Non-Financial Services	Japan	A	4
Non-Financial Services	Japan	BBB	11
Non-Financial Services	Japan	BB	3
Non-Financial Services	Japan	B	0
Non-Financial Services	Japan	CCC	0
Non-Financial Services	Asia ex-Japan	AA-AAA	3
Non-Financial Services	Asia ex-Japan	A	10
Non-Financial Services	Asia ex-Japan	BBB	7
Non-Financial Services	Asia ex-Japan	BB	3
Non-Financial Services	Asia ex-Japan	B	2
Non-Financial Services	Asia ex-Japan	CCC	0
Non-Financial Services	Rest of World	AA-AAA	0
Non-Financial Services	Rest of World	A	3
Non-Financial Services	Rest of World	BBB	1
Non-Financial Services	Rest of World	BB	1
Non-Financial Services	Rest of World	B	1
Non-Financial Services	Rest of World	CCC	0
Industrial Production	North America	AA-AAA	3
Industrial Production	North America	A	23
Industrial Production	North America	BBB	28
Industrial Production	North America	BB	15
Industrial Production	North America	B	9
Industrial Production	North America	CCC	3
Industrial Production	Europe	AA-AAA	0
Industrial Production	Europe	A	10
Industrial Production	Europe	BBB	22
Industrial Production	Europe	BB	14
Industrial Production	Europe	B	4
Industrial Production	Europe	CCC	0
Industrial Production	Japan	AA-AAA	0
Industrial Production	Japan	A	9
Industrial Production	Japan	BBB	18
Industrial Production	Japan	BB	6
Industrial Production	Japan	B	0
Industrial Production	Japan	CCC	0

Sector	Region	Rating	Num Obligors
Raw Materials	Rest of World	AA-AAA	0
Raw Materials	Rest of World	A	1
Raw Materials	Rest of World	BBB	4
Raw Materials	Rest of World	BB	1
Raw Materials	Rest of World	B	0
Raw Materials	Rest of World	CCC	0
Other Sectors	North America	AA-AAA	14
Other Sectors	North America	A	67
Other Sectors	North America	BBB	144
Other Sectors	North America	BB	43
Other Sectors	North America	B	35
Other Sectors	North America	CCC	6
Other Sectors	Europe	AA-AAA	22
Other Sectors	Europe	A	40
Other Sectors	Europe	BBB	81
Other Sectors	Europe	BB	19
Other Sectors	Europe	B	4
Other Sectors	Europe	CCC	2
Other Sectors	Japan	AA-AAA	14
Other Sectors	Japan	A	22
Other Sectors	Japan	BBB	23
Other Sectors	Japan	BB	9
Other Sectors	Japan	B	2
Other Sectors	Japan	CCC	0
Other Sectors	Asia ex-Japan	AA-AAA	9
Other Sectors	Asia ex-Japan	A	23
Other Sectors	Asia ex-Japan	BBB	24
Other Sectors	Asia ex-Japan	BB	6
Other Sectors	Asia ex-Japan	B	4
Other Sectors	Asia ex-Japan	CCC	0
Other Sectors	Rest of World	AA-AAA	3
Other Sectors	Rest of World	A	8
Other Sectors	Rest of World	BBB	17
Other Sectors	Rest of World	BB	8
Other Sectors	Rest of World	B	4
Other Sectors	Rest of World	CCC	1

3. The cross-section methodology

In the cross-section methodology, the proxy spread for a given obligor is the product of five factors:

(1) a global factor

(2) a factor for the industry sector of the obligor

(3) a factor for the region of the obligor

(4) a factor for the rating of the obligor

(5) a factor for the seniority of the obligor

In symbols, we can write the proxy spread of obligor i as:

$$S_i^{proxy} = M_{glob} M_{sctr(i)} M_{rgn(i)} M_{rtg(i)} M_{snty(i)}$$

Here *sctr(i), rgn(i), rtg(i)* and *snty(i)* denote respectively the sector, region, rating and seniority of obligor i. For example, for a senior unsecured claim on a European financial company rated BBB, we would have:

$$S_i^{proxy} = M_{glob} M_{FIN} M_{EUR} M_{BBB} M_{SEN}$$

The key assumption of this methodology is that there is a single multiplicative factor for (e.g.) all European obligors, independent of the sector, rating and seniority of those obligors. Similarly, there is a single multiplicative factor for all financial obligors, independent of the region, rating and seniority of those obligors.

This means that when we calibrate the proxy spread factors to liquid CDS spreads, we are using (for example) information from all BBB-rated obligors in calibrating M_{BBB}. Each factor is therefore represented by a reasonable number of obligors.

Calibration of the cross-section factors (M_{glob}, M_{FIN}, etc) to market data is straightforward, and proceeds as follows. If we number the factors from Global = 1 through the sectors, regions, ratings and seniorities, then we can write the model as:

$$y_i = \sum_{j=1}^{n} A_{ij} x_j$$

Here $y_i = \log(S_i^{proxy})$, $x_j = \log(M_j)$, and n is the number of factors (i.e. n is the total number of sectors, regions, ratings and seniorities, plus 1 for the global factor). A is a matrix of 1s and 0s, where A_{ij} is 1 if the sector, region, rating or seniority of obligor i is j, and 0 otherwise. Table [2] are a few rows of an example matrix A.

Table [2]: Cross-sectional *A* Matrix based on 18 February 2013 Markit data

Markit Ticker	Markit Short Name	Seniority	Global	Financials	Consumer Goods	Consumer Services	Industrials	Government	Utilities	Basic Materials	Energy	Telecommunications	Technology	Healthcare	North America	Europe	Japan	Asia ex-Japan	Australasia	Africa & Middle East	Latin America	AAA	AA	A	BBB	BB	B	CCC	Senior	Sub
AEP	Amern Elec Pwr Co Inc	Senior	1	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0
RABOBK	Rabobank Nederland	Sub	1	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	1
SNE	Sony Corp	Senior	1	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	1	0
SOAF	Rep South Africa	Senior	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	1	0

We want to find the optimal x that makes the proxy spreads S^{proxy} as close as possible to the market spreads S^{mkt}. Here we define "as close as possible" to mean "minimising total squared difference in log spreads", so finding the optimal x simply consists of performing a linear regression. Table [3] shows example spread factors, calibrated to Markit data from 18 February 2013. We also show the number of distinct liquid obligors in each category. In this calibration we used only obligors with at least three contributors.

Table [3]: Cross-sectional factors based on 18 February 2013 Markit data

	Spread Factor	Num Obligors
Global	151.2	1695
Financials	1.56	470
Consumer Goods	0.86	200
Consumer Services	0.93	186
Government	1.19	120
Industrials	0.94	186
Utilities	1.01	122
Basic Materials	1.03	111
Energy	1.00	107
Telecommunications	0.82	79
Technology	1.16	58
Healthcare	0.73	56
North America	0.84	724
Europe	1.05	552
Japan	0.82	179
Asia ex-Japan	0.90	129
Australasia	1.08	43
Latin America	1.26	34
Africa & Middle East	1.13	34
AAA	0.23	16
AA	0.42	102
A	0.48	465
BBB	0.80	685
BB	1.62	257
B	2.88	130
CCC	5.82	40
Senior	1	1551
Sub	1.10	144

4. Cross-section versus intersection

We now compare the cross-section and intersection methods, and explain the advantage of the cross-section method.

4.1 Empty buckets in the intersection method

The main problem with the intersection method is that typically there are sector/region/rating intersections containing few or no liquid obligors, making the proxy spread undefined. In table [1], we have used the broadest possible choice of sectors, regions and ratings that comply with the EBA proposal (apart from the addition of Japan as a region), but we still have a large number of empty (or nearly empty) buckets.

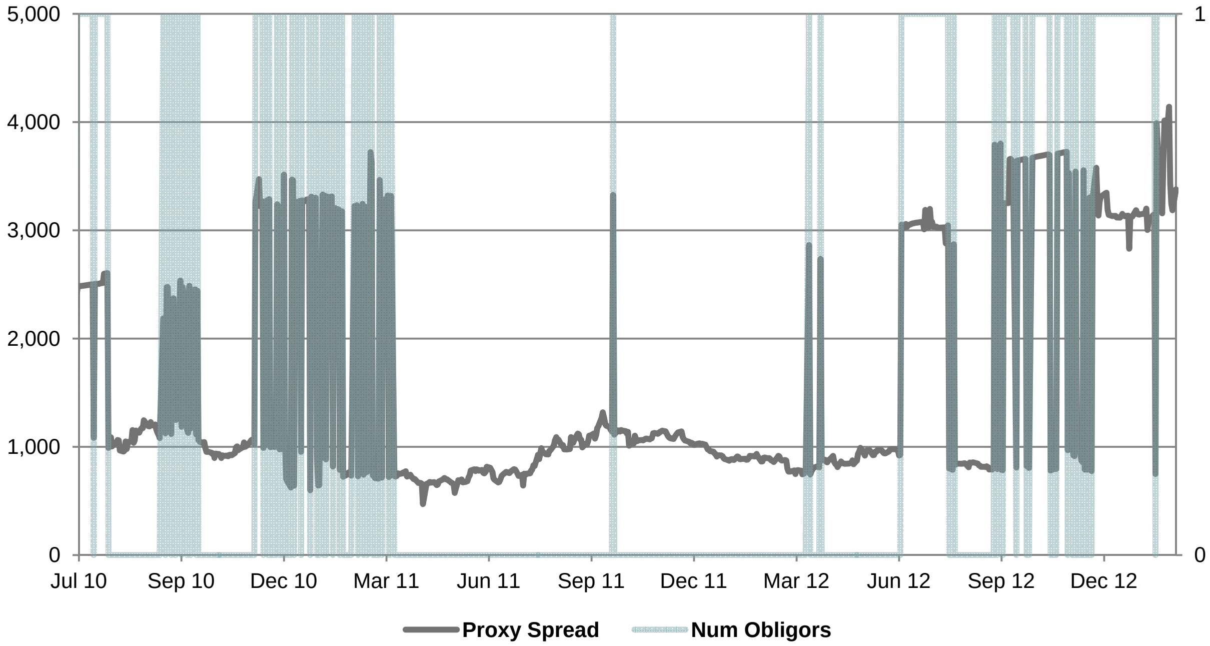
4.2 Historical Stability

Another problem with the intersection approach (and perhaps the most significant one from a practical perspective) is the historical behaviour of its proxy spreads. An obligor changing buckets (e.g. due to a rating migration) causes the proxy spreads of those buckets to jump. This will typically have the largest effect on buckets containing few liquid obligors. And if in addition this means one of the buckets switches between requiring and not requiring aggregation, the spread jump can be even more severe.

Graph [1] illustrates this problem with the example of the (Financial Services, Japan, B) bucket. The red line shows the number of obligors in the bucket, which has varied historically between 0 and 1, and the blue line shows the proxy spread. Since we have set the minimum number of obligors per bucket at 1, each time the bucket is empty we have to use aggregation to compute the proxy spread – in this case setting it to the average of all (Financial Services, B) liquid spreads. The one obligor in the bucket (Aiful Corp) has a spread which is typically much wider than the aggregated spread, so the proxy spread jumps dramatically each time Aiful Corp enters or leaves the bucket (which is mostly the result of days on which there were insufficiently many Markit contributors for that obligor).

Note that this is just one example of this problem – there are many other such examples.

Graph [1]: Proxy spread for B Japan Financials based on the intersection method



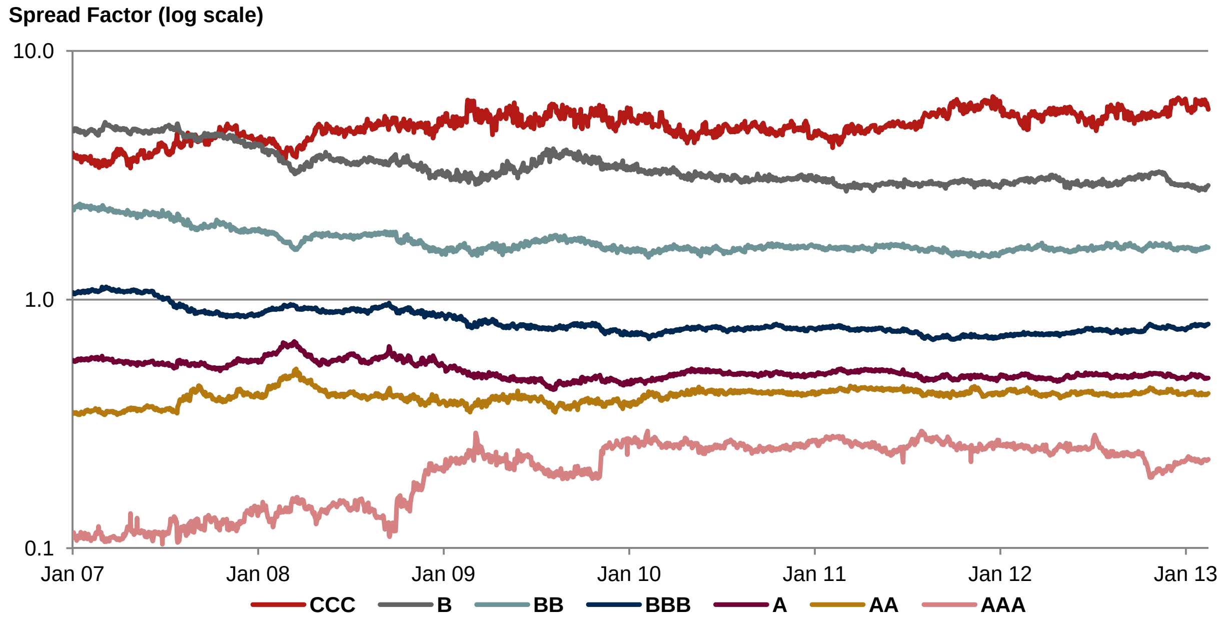
By contrast, the cross-section method has a proxy spread that is much more stable historically:

Graph [2]: Proxy spread for B Japan Financials based on the cross-section method

4.3 Monotonicity by Rating

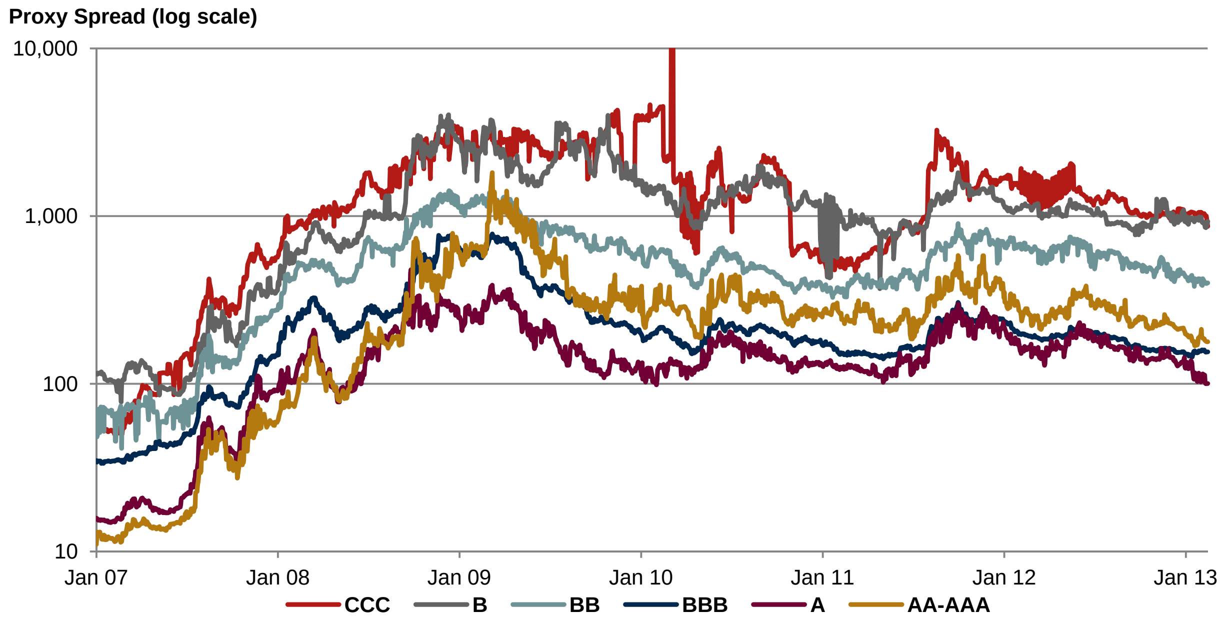
Out of rating, region and sector, rating is the strongest indicator of CDS spread. We can see from table [3] that the cross-section spread factors for 18 February 2013 are monotonic in rating – a worse rating has a wider proxy spread. And as graph [3] shows, this has almost always been the case historically.

Graph [3]: Historical credit spread factors for the cross-section method



By contrast, the intersection method is much less likely to produce monotonic proxy spreads by rating, as illustrated by graph [4], showing North American Financials proxy spreads. Notice in particular how the proxy spread for the top rating category (AA-AAA) is very often wider than those for both the A and BBB categories. Typically this is caused by one or two wide obligors which are rated AA- or better; because there are only a small number of other obligors in that bucket, the average spread is therefore rather wide. This behaviour is certainly counterintuitive and seems to indicate that the methodology is underperforming.

Graph [4]: Historical proxy spreads for North American Financials - Intersection Method



4.4 Granularity of Categories

When choosing sector, region and rating categories, there is a trade-off to be made. If the categories are too fine then there will be too few liquid obligors in some categories; if they are too broad then we lose potentially important information about the obligors.

With the cross-section method we have the flexibility to set the categories much finer than we could with the intersection method, since the cross-section method is much less likely to have sparsely-populated categories. This increases the accuracy of the cross-section method compared to the intersection method.

5. Conclusion

For calculating advanced CVA VaR capital requirement, financial institutions shall use a proxy spread level for illiquid counterparties that is appropriate having regard to the rating, region and industry of the counterparty. In this paper, we reviewed the intersection model proposed by the EBA that averages credit spread data of liquid names across the relevant rating, region, and sector sub-categories to imply the proxy spread of illiquid names. Unfortunately this method is dependent on data availability and quality. As a result, buckets with no or few constituents have un-defined points and unrealistic historical spread behaviour with jumps.

We propose an alternative approach based on a cross-section regression that has the advantage of providing more robust and stable results without loss of transparency. In contrast to the intersection model, the cross-section model generates realistic historic spread levels which could be used for CVA risk capital and P&L. The cross-section model is an easily implementable method that defines credit spreads for unobservable obligors based on rating, region, and sector. In addition to computing CVA risk as part of Basel 3, it can be used for computing P&L on CVA or illiquid credit products. Given its robustness over time, proxy time series can be implied and used for VaR and stressed VaR.

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