Dynamic modelling of single-name credits and CDO tranches

Martin Baxter
Fixed Income Quantitative Research
King’s College London, 6 June 2006
Outline of talk

- Levy processes
- Correlated Levy processes
- Credit modelling
- Single-names
- CDO portfolios
- Parameters
Motivation and intuition

- Credit is about extreme events
- Extreme events through jumps
- Jumps can be global or idiosyncratic
- Need a dynamic model, arbitrage-free
- Tractable
Levy processes

- Poisson process – jumps of size 1 occur at rate $\lambda$
- Levy process – jumps of size $x$ occur at rate $\nu(x)dx$
- Formally $X(t)$ is a Levy process if it has stationary independent increments, that is
  - $X(s+t) - X(s)$ is independent of $(X(u): u \leq s)$, and is distributed as $X(t)$.
- All Levy processes are made up of drift, Brownian motion and jumps

$$E(\exp(\theta X_t)) = \exp(t \psi(\theta))$$

$$\psi(\theta) = \mu \theta + \frac{1}{2} \sigma^2 \theta^2 + \int (e^{\theta x} - 1) \nu(x) dx$$

- Drift $\mu$, Brownian volatility $\sigma$, jump measure $\nu$. 
Gamma process

- **Gamma process** – pure jump increasing process with jump measure
  \[ \nu(x) = \gamma x^{-1} \exp(-\lambda x) \]
- Marginal distribution is the continuous gamma distribution.
- Parameter \( \gamma \) is jump intensity
- Parameter \( \lambda \) is inverse jump size
- Notation \( X(t) = \Gamma(t; \gamma, \lambda) \)

- **Variance Gamma process** (Moosbrucker) – difference of two Gammas
  \[ X(t) = VG(t; \gamma, \lambda, \lambda_u) = \Gamma(t; \gamma, \lambda_u) - \Gamma(t; \gamma, \lambda) \]
Correlated Levy processes

- Start with any Levy process distribution $X(t)$.
- Make independent copies $X_g$ (global factor) and $\tilde{X}_1, \ldots, \tilde{X}_n$ (idiosyncratic factors)
- Then define correlated Levy processes as

$$X_i(t) = X_g(\phi t) + \tilde{X}_i((1-\phi)t)$$

- So that each $X(i)$ has the same distribution as $X$, and the correlation between $X(i)$ and $X(j)$ is $\phi$. 
Linking back to Gaussian copula

- The Gaussian copula can be thought of as

\[ X_i = \sqrt{\rho} Z_g + \sqrt{1-\rho} Z_i \]

- This can be made dynamic as

\[ X_i(t) = \sqrt{\rho} W_g(t) + \sqrt{1-\rho} W_i(t) \]

- And then by the space-time equivalence of BM

\[ X_i(t) = W_g(\rho t) + W_i((1-\rho)t) \]

- This now matches our general form.
Credit modelling – basic Gamma model

- Structural model with value of the firm $S(t)$
  
  $$S_t = S_0 \exp(-\Gamma(t; \gamma, \lambda) + \mu t), \quad \mu = \gamma \log(1 + \lambda^{-1})$$

- Assume $S(t)$ is a positive martingale with up-drift and down-jumps

- Entity defaults when $S(t)$ goes below a threshold

- Correlation structure as before

- Reduces to log-normal as $\gamma$ goes to infinity

- “No news is good news”
Credit modelling – extra features

- Add “good news” jumps with Variance Gamma instead of Gamma (VG).

- Add “catastrophe” term with extra low-intensity high-impact global factor (CG).

- Add Brownian term to get continuous random movement as well (BG).

- Also various combinations, such as Brownian-Variance-Gamma (BVG), Catastrophe-Variance-Gamma (CVG), etc.
Credit modelling – sample path

Stock-price path

- Gamma model
- Brownian Gamma
Single-name credit dynamics for Gamma

- Can fit $\gamma$, $\lambda$ to basket of credits, such as CDX 125 S6.
- Average spread error is only 6bp (at 19 April 2006)
- Median case, Devon Energy Corp, is shown
CDO Pricing

- We can reformulate the model as

\[ X_t^i = -\Gamma_g(t; \phi \gamma, \lambda) - \Gamma_i(t; (1 - \phi) \gamma, \lambda) \]

- Decomposes the log-value into global and idiosyncratic Gamma terms

- Default by \( t \), if \( X^i(t) \) below a threshold (European approximation)

- Lambda parameter is redundant due to scaling

- Phi parameter moves spread from junior to senior

- Gamma parameter moves spread from equity/senior to mezzanine
### Fitting to CDO prices (CDX)

<table>
<thead>
<tr>
<th>Tranche</th>
<th>5y CDX</th>
<th>7y CDX</th>
<th>10y CDX</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Market</td>
<td>Model</td>
<td>Market</td>
</tr>
<tr>
<td>0% - 3%</td>
<td>1,333.8</td>
<td>1,334.5</td>
<td>1,679.0</td>
</tr>
<tr>
<td>3% - 7%</td>
<td>96.5</td>
<td>96.3</td>
<td>240.0</td>
</tr>
<tr>
<td>7% - 10%</td>
<td>19.5</td>
<td>25.3</td>
<td>44.5</td>
</tr>
<tr>
<td>10% - 15%</td>
<td>10.0</td>
<td>12.2</td>
<td>19.5</td>
</tr>
<tr>
<td>15% - 30%</td>
<td>5.0</td>
<td>4.1</td>
<td>7.0</td>
</tr>
<tr>
<td>30% - 100%</td>
<td>3.1</td>
<td>0.3</td>
<td>4.0</td>
</tr>
</tbody>
</table>

**Best fit score (bp)**

<table>
<thead>
<tr>
<th></th>
<th>5y CDX</th>
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<th>10y CDX</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2.8</td>
<td>4.1</td>
<td>9.7</td>
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Fitting as at 2 June 2006.

<table>
<thead>
<tr>
<th>Maturity</th>
<th>Gamma</th>
<th>Phi</th>
</tr>
</thead>
<tbody>
<tr>
<td>5y</td>
<td>56.0%</td>
<td>9.3%</td>
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<tr>
<td>7y</td>
<td>13.9%</td>
<td>13.8%</td>
</tr>
<tr>
<td>10y</td>
<td>15.4%</td>
<td>13.6%</td>
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</table>
Fitting to CDO prices (iTraxx)

<table>
<thead>
<tr>
<th>Tranche</th>
<th>5y iTraxx</th>
<th>7y iTraxx</th>
<th>10y iTraxx</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Market</td>
<td>Model</td>
<td>Market</td>
</tr>
<tr>
<td>0% - 3%</td>
<td>1,075.6</td>
<td>1,076.0</td>
<td>1,391.7</td>
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<tr>
<td>3% - 6%</td>
<td>69.5</td>
<td>69.3</td>
<td>185.5</td>
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<tr>
<td>6% - 9%</td>
<td>19.0</td>
<td>21.1</td>
<td>46.0</td>
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<tr>
<td>9% - 12%</td>
<td>8.7</td>
<td>11.0</td>
<td>25.0</td>
</tr>
<tr>
<td>12% - 22%</td>
<td>3.8</td>
<td>4.7</td>
<td>8.3</td>
</tr>
<tr>
<td>22% - 100%</td>
<td>1.7</td>
<td>0.4</td>
<td>3.3</td>
</tr>
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<td>Best fit score (bp)</td>
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<tr>
<td>5y</td>
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<tr>
<td>7y</td>
<td>24.3%</td>
<td>12.2%</td>
</tr>
<tr>
<td>10y</td>
<td>19.9%</td>
<td>19.6%</td>
</tr>
</tbody>
</table>
## Model comparison

<table>
<thead>
<tr>
<th>Model</th>
<th>CDX 5y</th>
<th>CDX 7y</th>
<th>CDX 10y</th>
<th>iTraxx 5y</th>
<th>iTraxx 7y</th>
<th>iTraxx 10y</th>
<th>Average (bp)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cat Gamma</td>
<td>1.4</td>
<td>7.9</td>
<td>15.4</td>
<td>1.1</td>
<td>7.0</td>
<td>8.7</td>
<td>6.9</td>
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<tr>
<td>Variance Gamma</td>
<td>2.9</td>
<td>9.6</td>
<td>15.7</td>
<td>2.9</td>
<td>9.6</td>
<td>7.0</td>
<td>8.0</td>
</tr>
<tr>
<td><strong>Gamma</strong></td>
<td><strong>3.3</strong></td>
<td><strong>7.7</strong></td>
<td><strong>17.2</strong></td>
<td><strong>3.2</strong></td>
<td><strong>6.8</strong></td>
<td><strong>17.0</strong></td>
<td><strong>9.2</strong></td>
</tr>
<tr>
<td>Brownian Gamma</td>
<td>4.7</td>
<td>11.1</td>
<td>18.3</td>
<td>3.9</td>
<td>9.2</td>
<td>13.8</td>
<td>10.2</td>
</tr>
<tr>
<td>Brownian VG</td>
<td>2.8</td>
<td>21.9</td>
<td>44.2</td>
<td>2.3</td>
<td>18.2</td>
<td>40.6</td>
<td>21.7</td>
</tr>
<tr>
<td>Cat VG</td>
<td>1.4</td>
<td>28.6</td>
<td>48.1</td>
<td>1.0</td>
<td>26.4</td>
<td>34.7</td>
<td>23.4</td>
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<tr>
<td>Global Info Time</td>
<td>77.0</td>
<td>28.5</td>
<td>66.1</td>
<td>38.0</td>
<td>32.3</td>
<td>49.5</td>
<td>48.6</td>
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<tr>
<td>Gaussian copula</td>
<td>38.9</td>
<td>66.1</td>
<td>76.3</td>
<td>33.6</td>
<td>75.7</td>
<td>83.9</td>
<td>62.4</td>
</tr>
</tbody>
</table>

Both Gamma and its simple variants (CG, VG, and BG) do well.

BVG and CVG have symmetric up and down jumps which are not as good.

Data weekly observed from 12 October 2005 to 5 April 2006.
Parameters (G) – Gamma

Gamma parameter

- CDX 5y
- CDX 7y
- CDX 10y
- iTraxx 5y
- iTraxx 7y
- iTraxx 10y
Parameters (G) - Phi

![Graph showing Phi parameter over time for different indices such as CDX 5y, CDX 7y, CDX 10y, iTraxx 5y, iTraxx 7y, and iTraxx 10y. The graph plots the Phi parameter against dates from September 2005 to May 2006.]
Parameters (VG) – Lambda Up

Variance Gamma model: Lambda-Up parameter

- CDX 5y
- CDX 7y
- CDX 10y
- iTraxx 5y
- iTraxx 7y
- iTraxx 10y
Summary

- Levy processes create a family of models for credit
  - Jumps to match market tails
  - Simple correlation structure
- Single name dynamics consistent with CDS market
- Multivariate dynamics close to market CDO prices
- Simple Gamma model has good properties
  - Intuitive model
  - Acceptable fit
  - Stable parameters, not too many
  - Tractable to implement
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- **Credit Derivatives**

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